Multiple strategies for finding ratio of two variables in an equation: A Learning Study

LEUNG, Chi-keung Eddie
YEUNG, Sen Yan Ryan

Hong Kong Insitute of Education

Introduction

- *a learning study conducted in a secondary school of high level of achievement*
- *school teachers suggested that ratio is a challenging topic for Secondary 1 (S1) students of age around twelve*
- a systematic process of inquiry into lesson teaching and learning

conceptual framework builds on three types of variation

- V1: Variation in students' different ways of understanding of the topic
- V2: Variation in the teachers' understanding and way of dealing with the object of learning;
- V3: Variation used as a guiding principle for pedagogical design

V2: Starts with Variation in teachers' understanding of what the most worthwhile object of learning is & ways of handling it

- Many students are weak in simplifying ratios with fractions
- $\frac{2}{3} : \frac{1}{2}$ = 4:3

- Many students are confused with ratios' quantities.

e.g. they could not tell clearly the ratio of a:b in 3a=2b is a:b = 3: 2 or a:b = 2:3

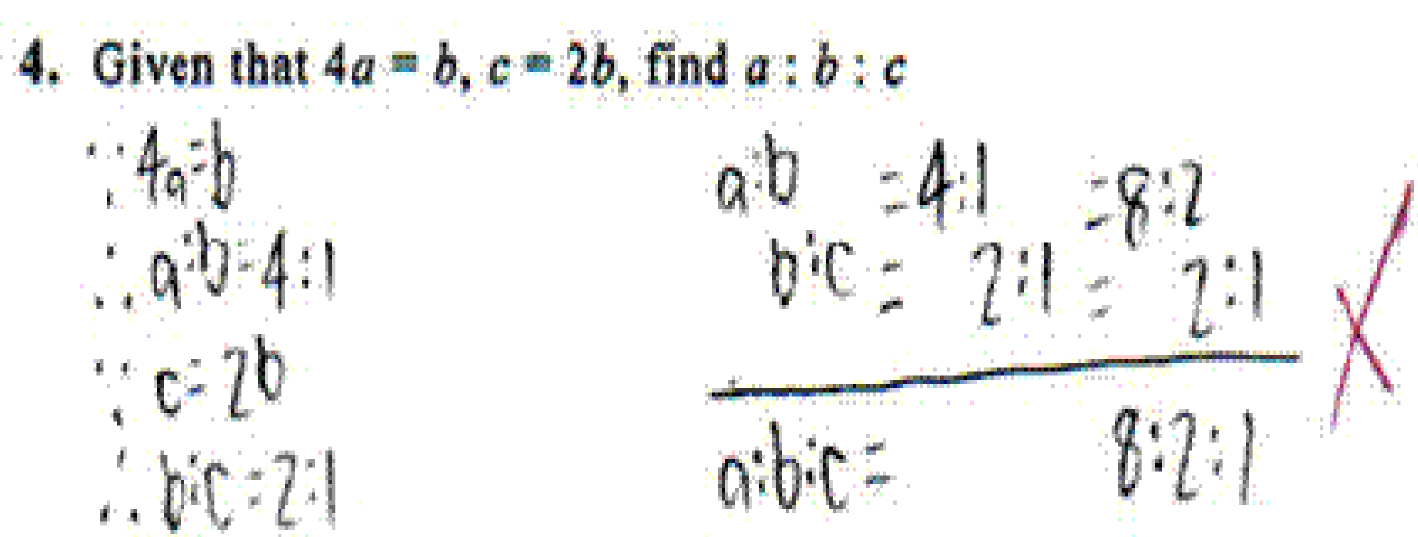
- Students are weak in combining 2 two-term ratio into three- term ratio

e.g. a:b , b:c → a:b:c

V1: The identification of learning difficulties

- A diagnostic test was set and given to the S2 students
- to reveal persistent preconceptions and possible difficulties that presented themselves as obstacles for learning

students were procedurally correct
but less satisfactory in finding the ratio of
two variables in an equation (e.g. 4a = b)


4. Given that $4a = b$, $c = 2b$, find $a : b : c$

$\because 4a = b$

$\therefore a : b = 4 : 1$

$\because c = 2b$

$\therefore b : c = 2 : 1$

$a : b = 4 : 1 = 8 : 2$

$b : c = 2 : 1 = 2 : 1$

$a : b : c = 8 : 2 : 1$

Interview with students for better understanding of their views

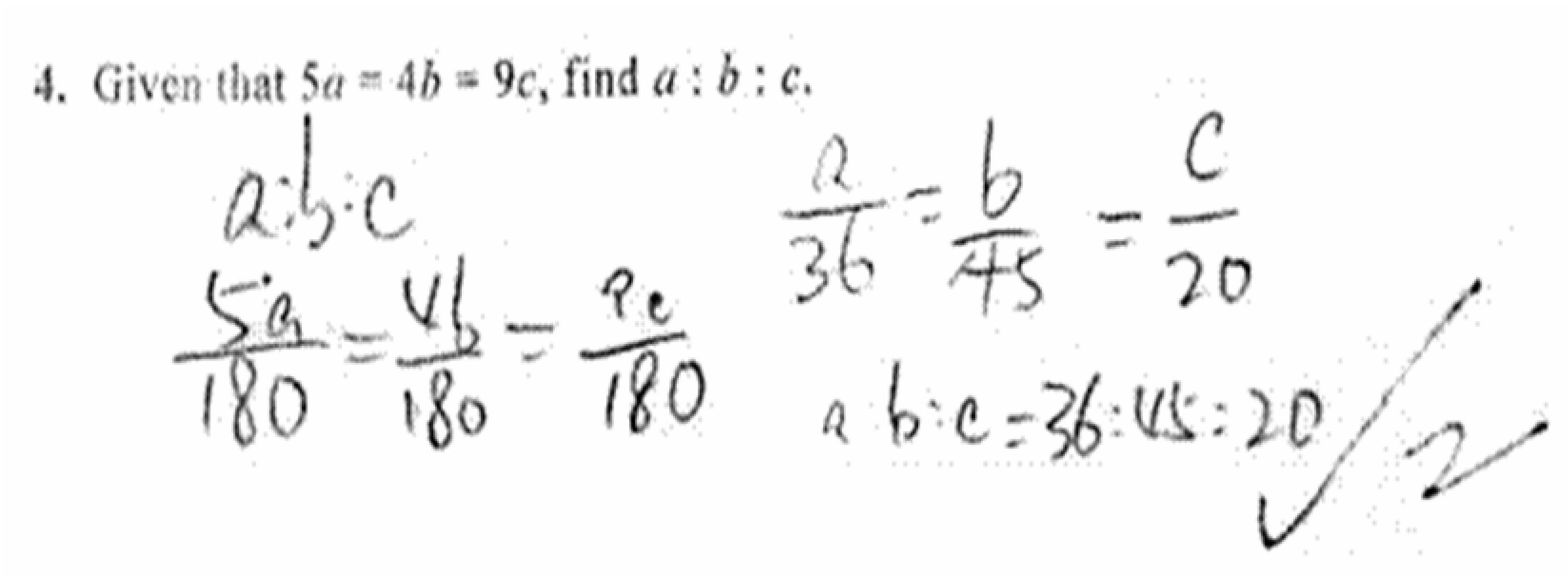
- Researcher: What is the ratio of a: b in the equation $3a=b$.
- Student A (ave ability): It should be 3 : 1.
- Researcher: How did you get it?
- Student A: Three *a* is equal to one *b*. Therefore, the quantity of *a* has 3parts and the quantity of *b* has 1 part.
- (viewed variable as a quantity and were confused with the coefficient of the variable with the number of parts of the variable being divided)

- Researcher: What is the ratio of a: b in the equation 3*a*=*b*.
- Student B (low ability): I think it is 3 : 1.
- Researcher: How did you get it?
- Student B: I just guessed.
- Researcher: O.K. What do you know about 3:5 ?
- Student B: 3:5 can be written as 3/5
- Researcher: Let’s take a look at the equation of 3*a*=*b* again*.* Can you find out the ratio of *a*:*b* by finding a/b through algebraic manipulations of 3*a*=*b* ?
- Student B: No, I couldn’t.

- Researcher: What is the ratio of *b*:*c* in the equation 4*c*=3*b*
- Student C: The ratio of *b*:*c* should be 4:3
- Researcher: How did you get it?
- Student C: At first, I felt that the answer was either 4:3 or 3:4. Then I divided a paper separately into four parts (representing 4c) and three parts (representing 3b). I could see b was larger than c. So, I think the ratio of b:c should be 4:3.
- (use concrete representation of 4*c*=3*b* to help identify the larger quantity and find out the correct ratio)

Many got the right answer through algebraic manipulation

- One simple and elegant answer

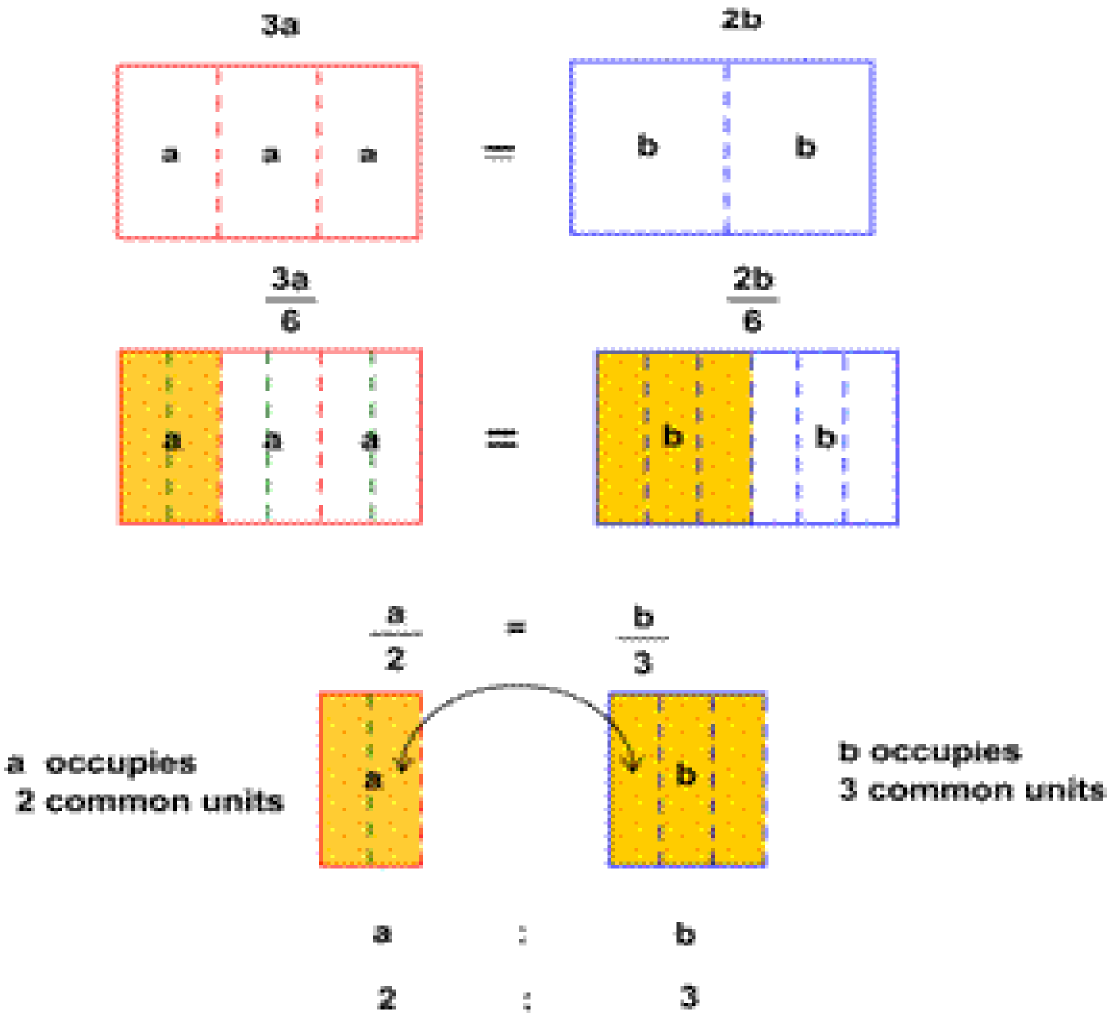


Object of learning:
To find the ratio of two quantities from an equation of the form ax=by where *a*, *b* are constant

- Critical aspects:
- CF1: To identify the larger quantity
- CF2: two quantities in ratio form is based on the common unit.
- CF3: the ratio is unaltered if the ratio are both multiplied or divided by the same factor
- CF4: ratio of two quantities can be expressed as fraction

V3: Lesson Planning & the Use of Variation

- **Method 1: Use LCM to find the ratio of two variables in an equation**
- An equation $3a = 2b$ was given, students worked in pair and fold a paper to represent $3a$ and $2b$ respectively
- Which one is larger, a or b?
- How to find $a:b$?
- (learning through “concrete” experience)



3a
2b
a a a
=
b b
3a/6
2b/6
a a a
=
b b
a/2
=
b/3
a occupies 2 common units
a
b
b occupies 3 common units
a : b
2 : 3

- Some guiding questions:

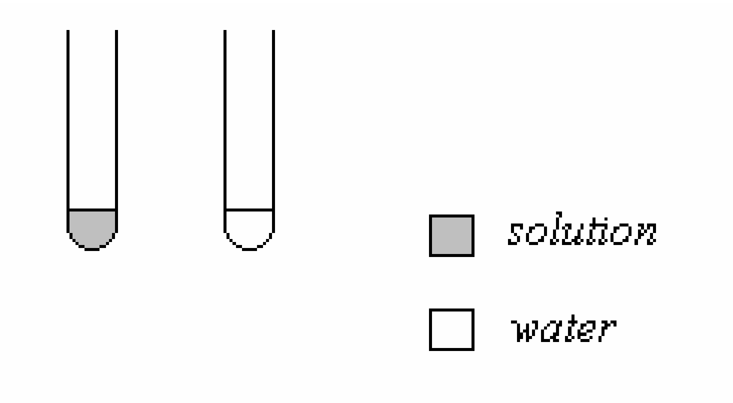
- How to compare a and b?
- How can we make a common unit for comparing a and b?
- What is “6”? Can it be 5?

Use of Variation

Invariant	Variation	Critical feature to be discerned
Size of the paper is equal Both a and b share a common unit	Amount of common unit.	CF1: To identify which quantity is large (*a* or *b* ?) CF2: the ratio of two quantities is based on a common unit.

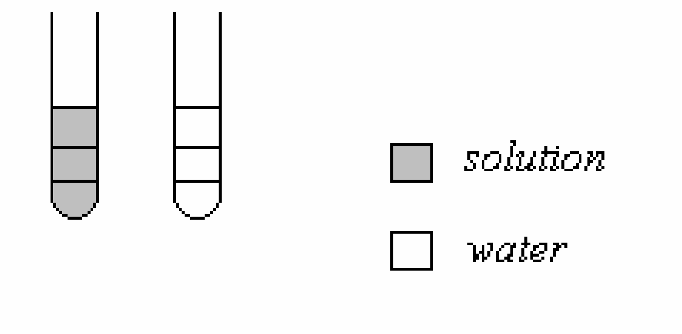
Method 2: Use substitution to find the ratio of two variables in an equation

- Teacher mixes 1 portion of ribena to 1 portion of water and let the students to observe the color.



Showing 3 portions of ribena, teacher asked how much water will be

Needed to get the same color intensity as the previous one



- Teacher guided students to understand $x : y = 3x : 3y$ and that the ratio doesn't change when all parts are multiplied by the same factor
- Can substitution help find $a{:}b$ in $3a = 4b$?
- How?

Method 1

$a : b = 3a : 3b$

$= 4b : 3b$

$= 4 : 3$

Method2

$a : b = 4a : 4b$

$= 4a : 3\mathrm{a}$

$= 4 : 3$

Use of Variation

Invariant	Variation	Critical feature to be discerned
The color density	Volume/portion of Ribena and water	CF3 : To understand the ratio doesn’t change when the components are multiplied by the same factor

Method 3: Use algebraic method to find the ratio

- Teacher guided students to recall that the ratio of a to b can be expressed as a/b
- Given an equation $6x = 5y$, students were invited to find the ratio through algebraic manipulations of fraction

Use of Variation

Invariant	Variation	Critical feature to be discerned
equation	Approach to find the ratio	CF4: To find the ratio through algebraic manipulations of fraction

Implementation of the research lesson

- research lesson was implemented in three cycles (1A, 1D, 1E)
- In each cycle, one of the three participant teachers taught the research lesson to his or her class while other teachers and research team members observed
- Each cycle of the research lesson was followed by a post-lesson conference in which the group members shared their insights and observations made during the lesson

Teaching in the Research Lesson and Recommend Changes

- When dealing with Method 1, the teachers focused on relating the “6” with the LCM of 2 and 3
- students were still not very clear about the common units shared by the two quantities in the first and second cycles
- Change: highlight the common unit

Dealing with Method 2 for $3a = 4b$

- students were inclined to copy the teacher's demonstration
- Able to write $a : b = 3a : 3b$
- Not certain to substitute $4b$ into $3a$
- Did not understand clearly the aims of multiplying the ratio by 3
- Change: guide students to think why multiplied by 3? Would other numbers also work for this example?

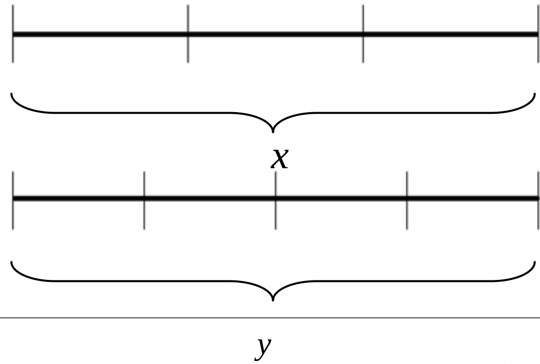
Dealing with Method 3 for $6x = 5y$

- Students were asked to divide the both sides by 6, *x*, 5 and *y* alternately and to investigate how to get the fraction x/y

- Change: students were allowed to choose other combinations for division in the third cycle
- Observed that some students could divide the both sides by 6*y* in order to get x/y

Comparison of the pre- & post-test results

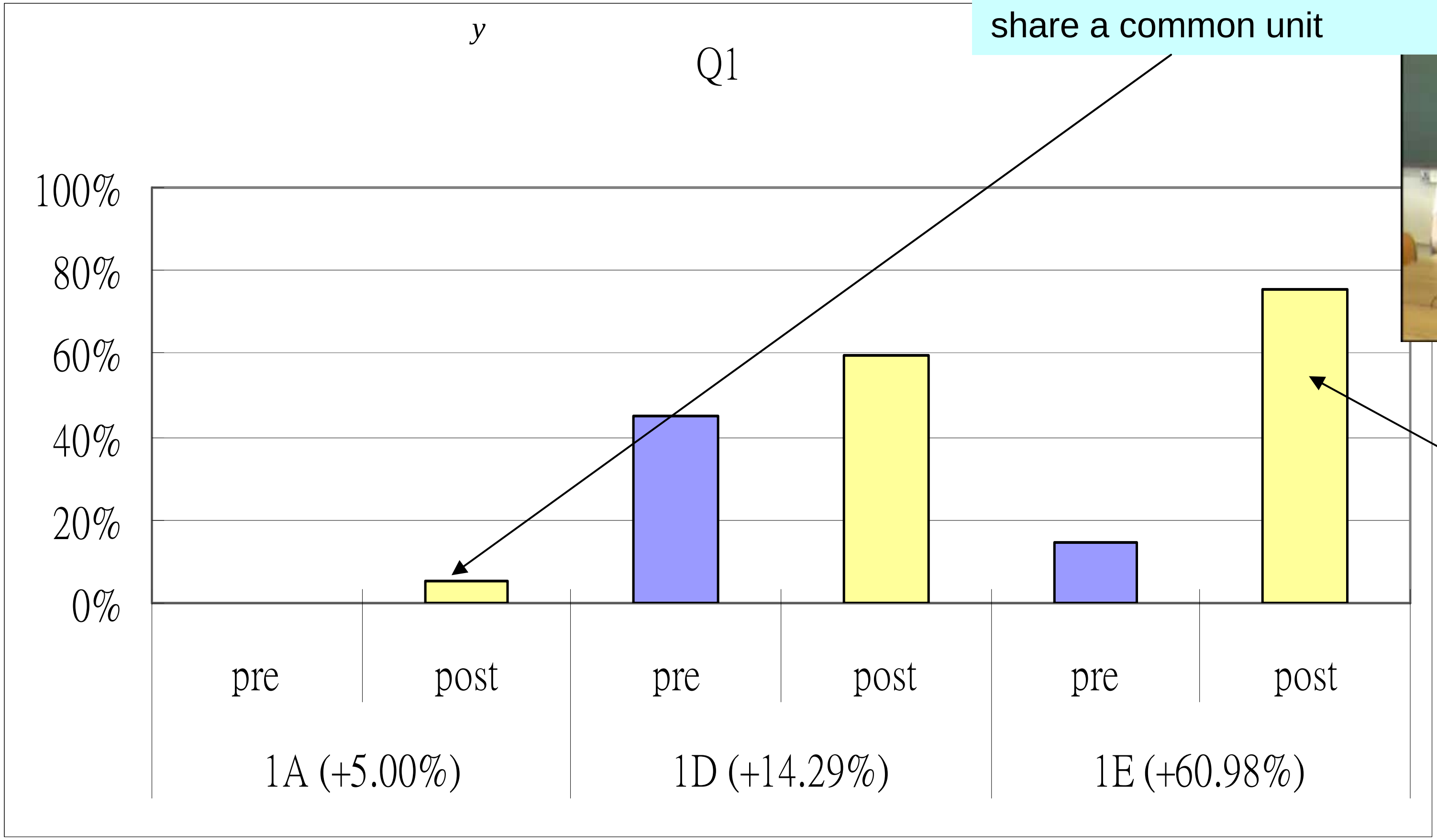
1. Look at the diagram below and write down the ratio *x* : *y*.



x : y = **1:1**

Student Interview

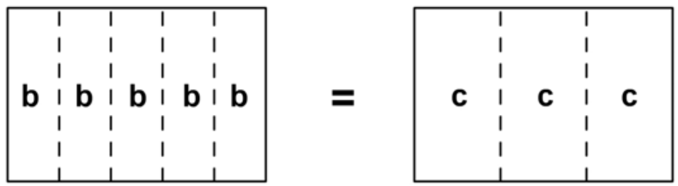
Students do not know that a:b share a common unit



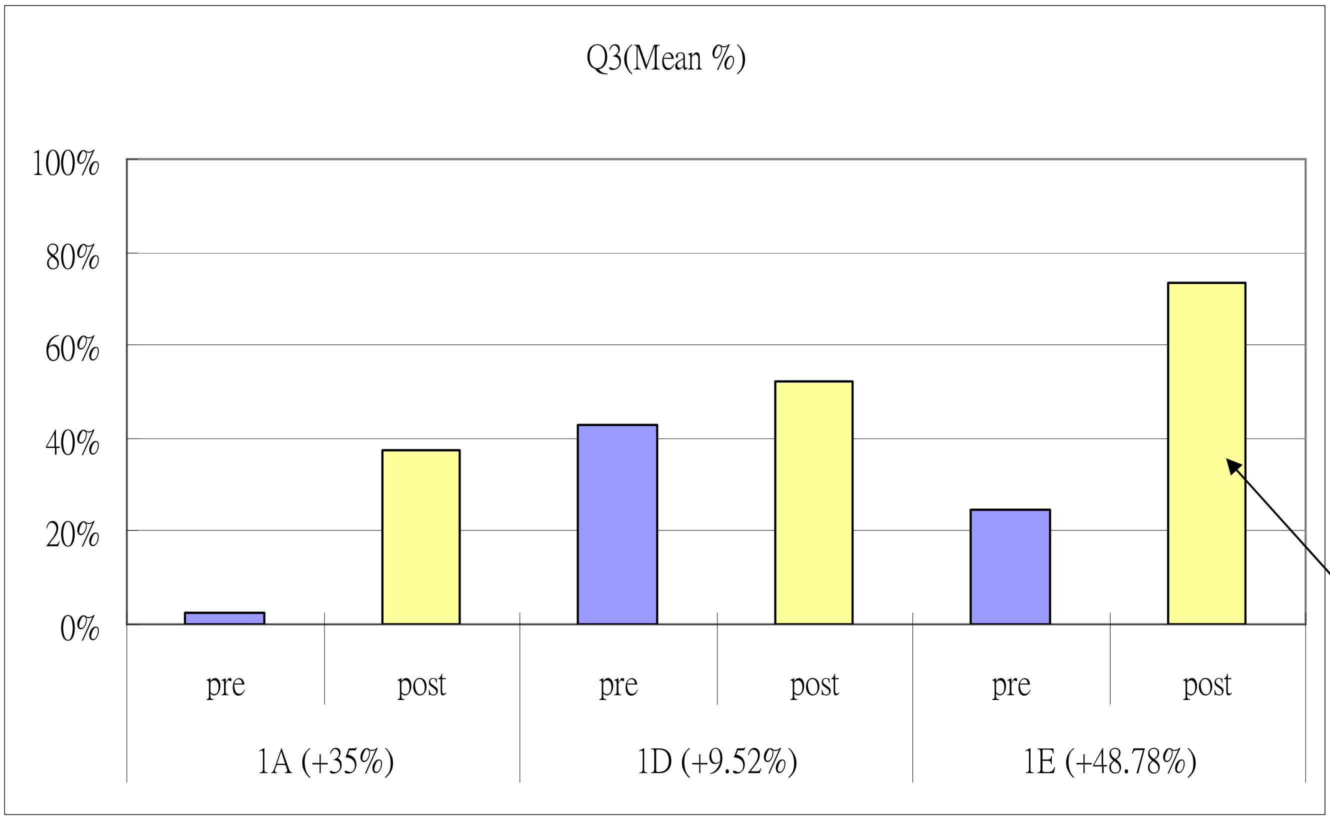
Student Interview

Comparison of the pre- & post-test results

3. Given



Find *b* : *c*



Student Interview

Student can transform into the equation

5b = 3c in order to find the rato b:c

Comparison of the pre- & post-test results

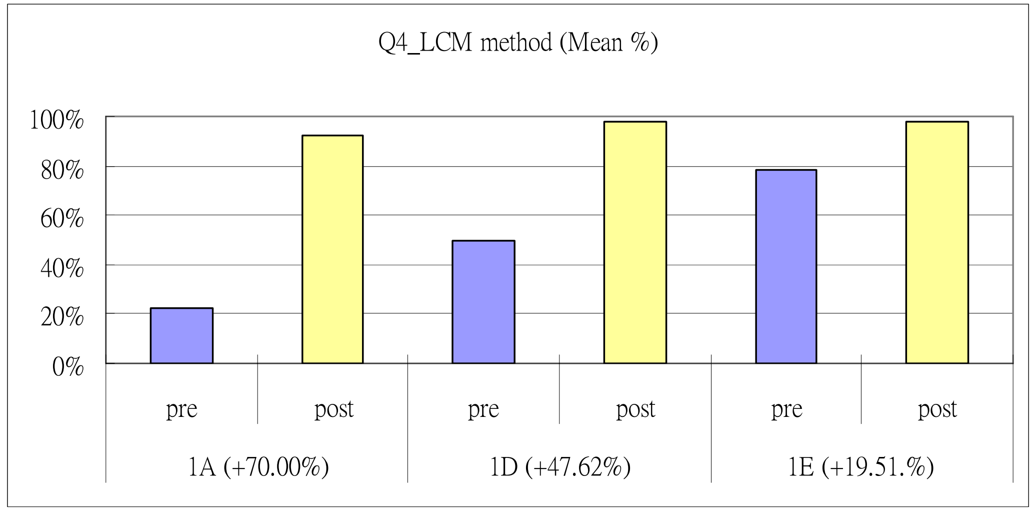
4. Find *a* : *b* by filling the blanks

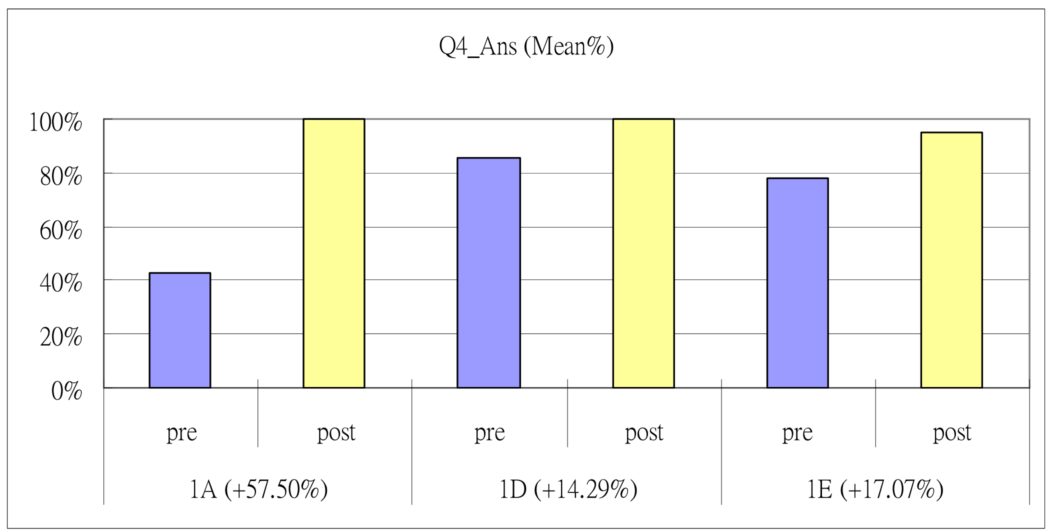
$$7a = 8b$$

$$\frac{7a}{\square} = \frac{8b}{\square}$$

$$\frac{a}{\square} = \frac{b}{\square}$$

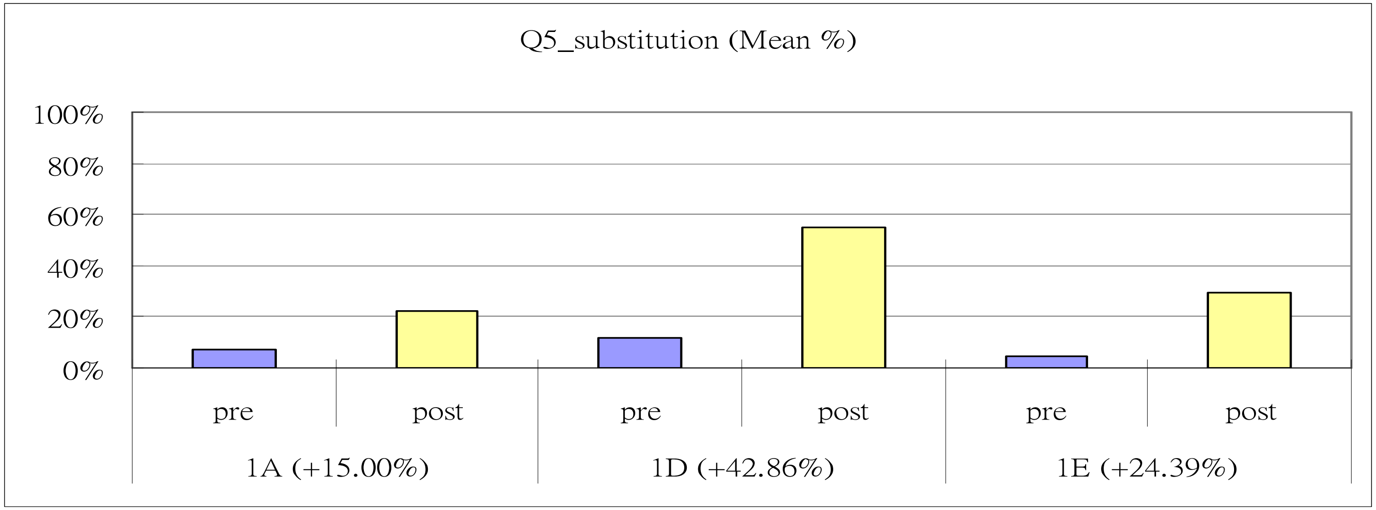
$$a : b = \square : \square$$

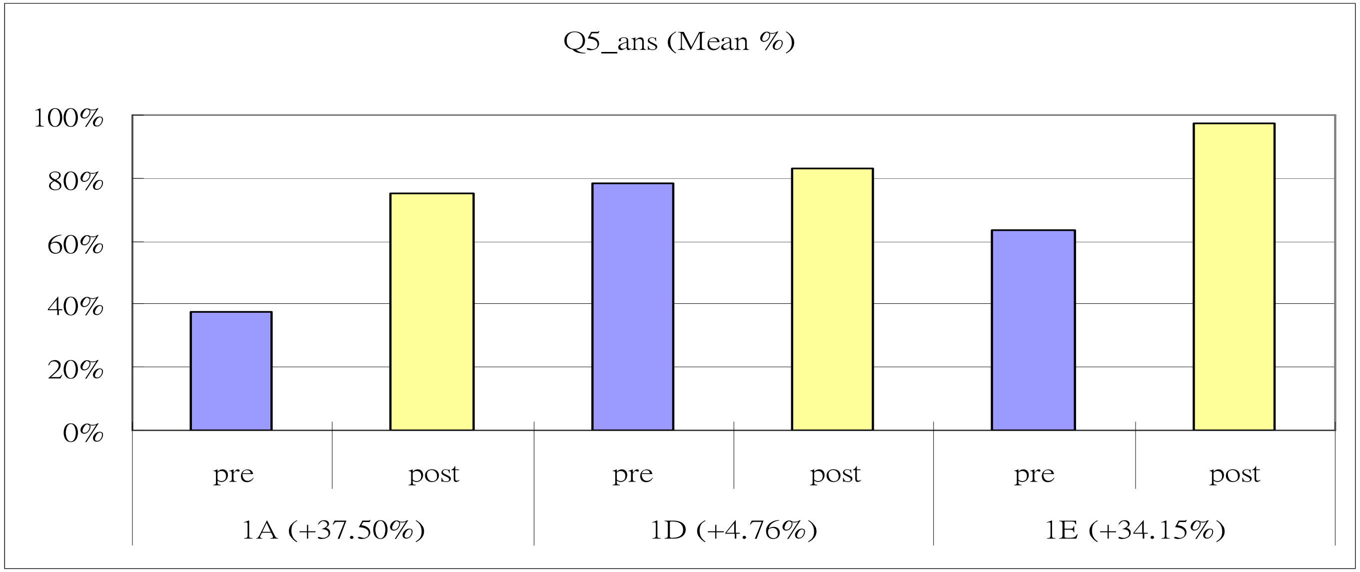




Comparison of the pre- & post-test results

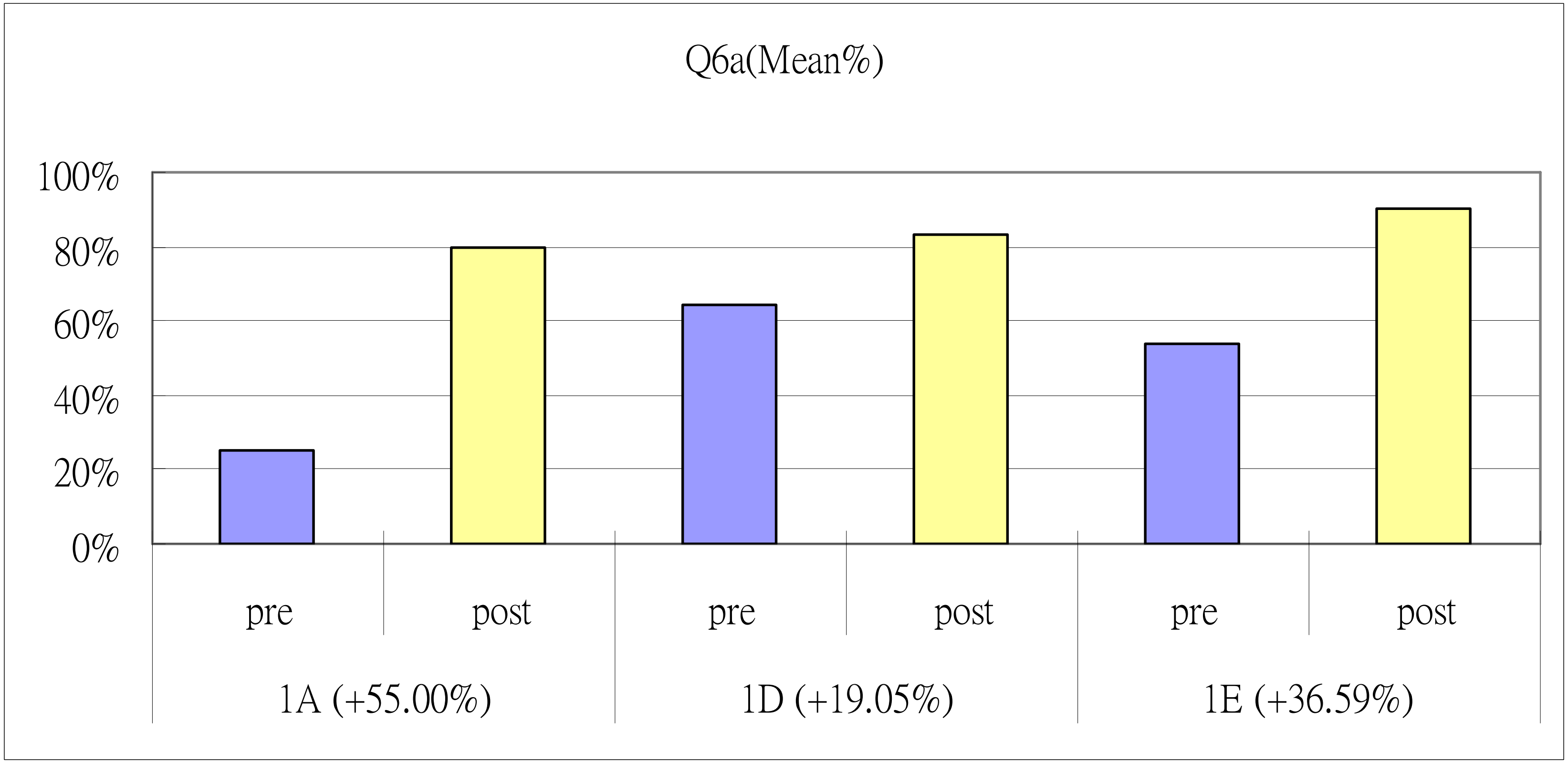
5. Given $8a = 12b$, find $8a : 8b$.





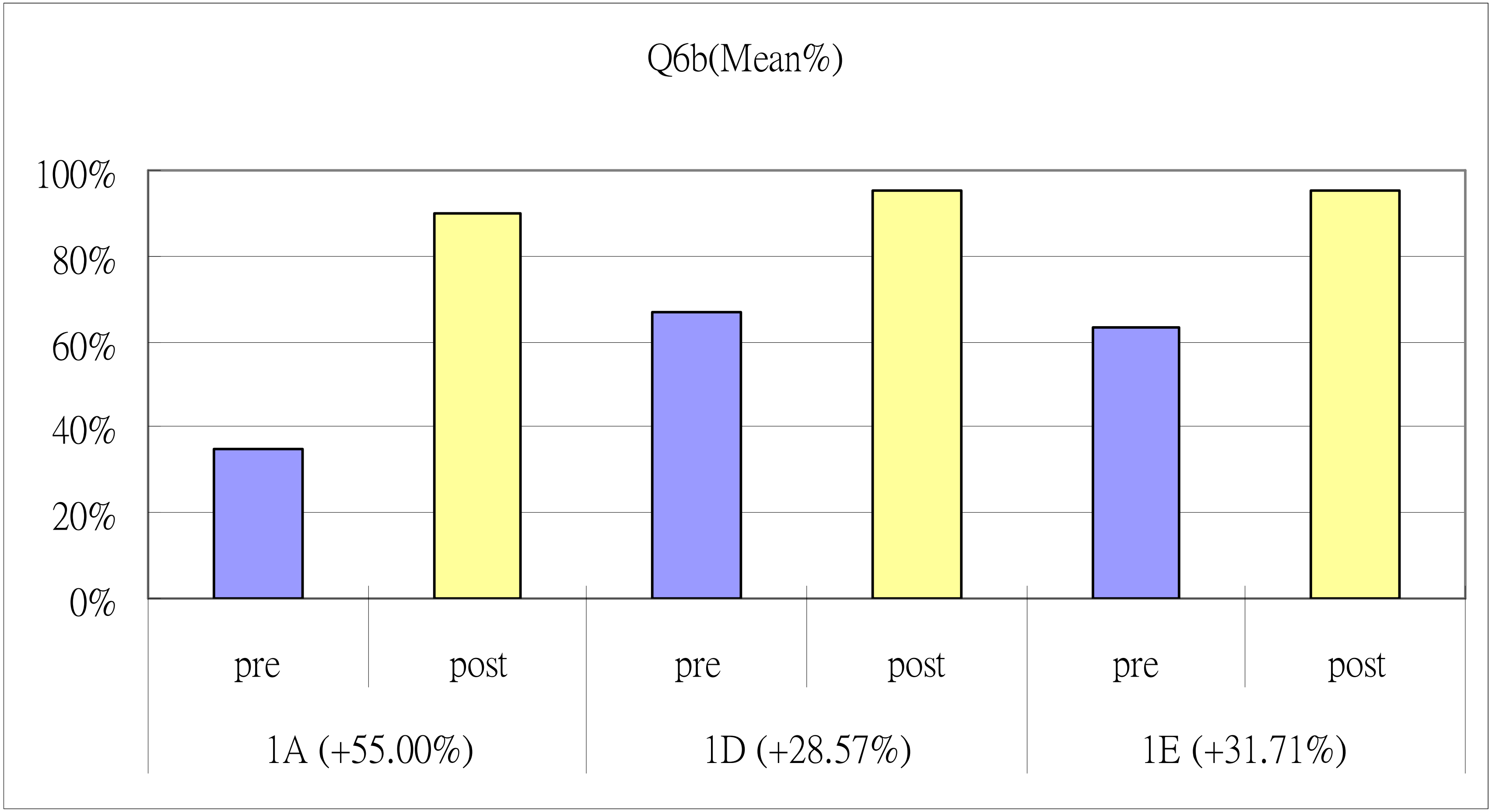
Comparison of the pre- & post-test results

6. Given $5x = 16y$, find (a) $\frac{x}{y}$ (b) $x : y$



Comparison of the pre- & post-test results

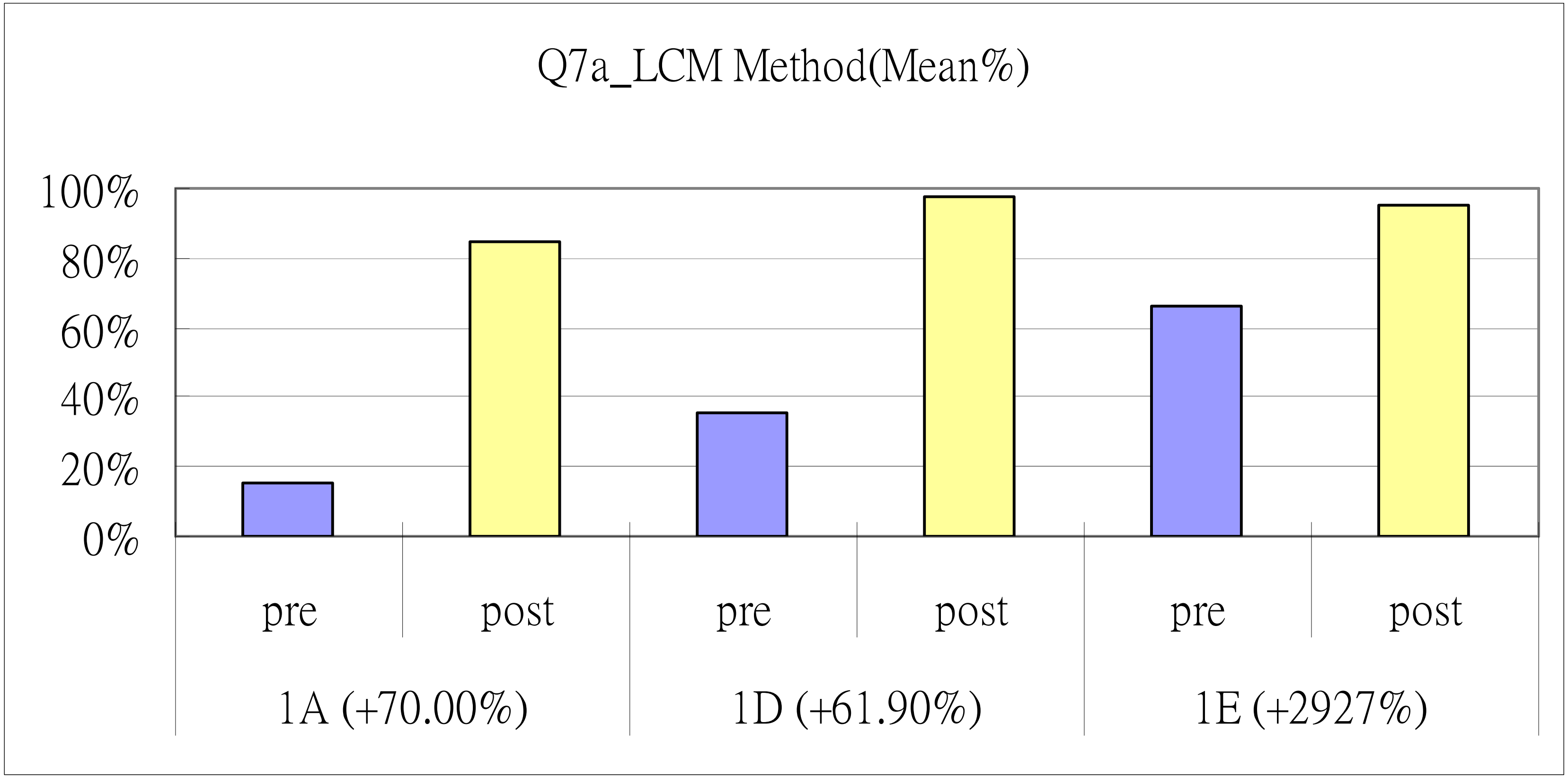
6. Given $5x = 16y$, find (b) $x : y$



Comparison of the pre- & post-test results

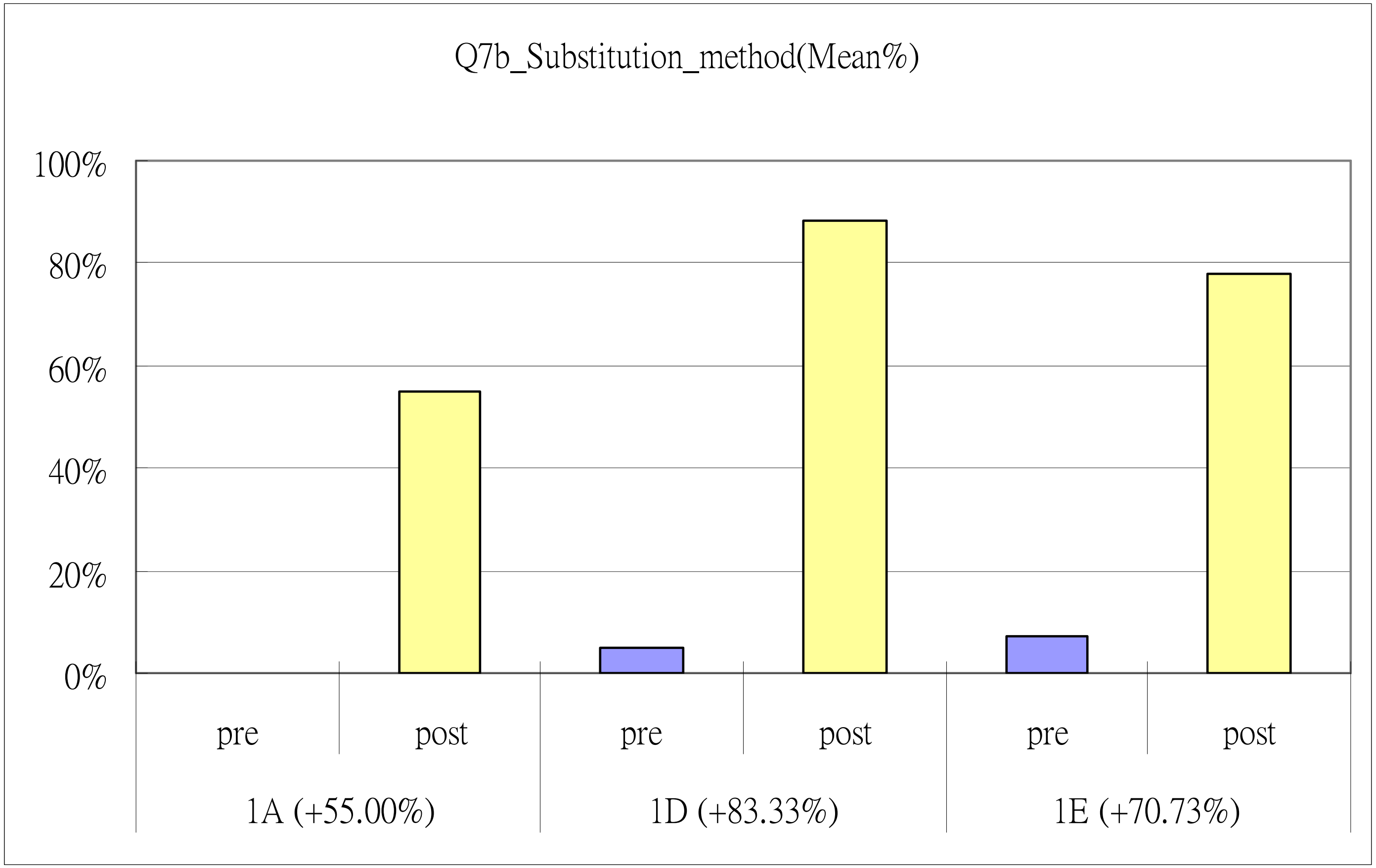
7. Given that 3*b* = 8*c*, find *b* : *c* by using the following methods:

(a) L.C.M.



Comparison of the pre- & post-test results

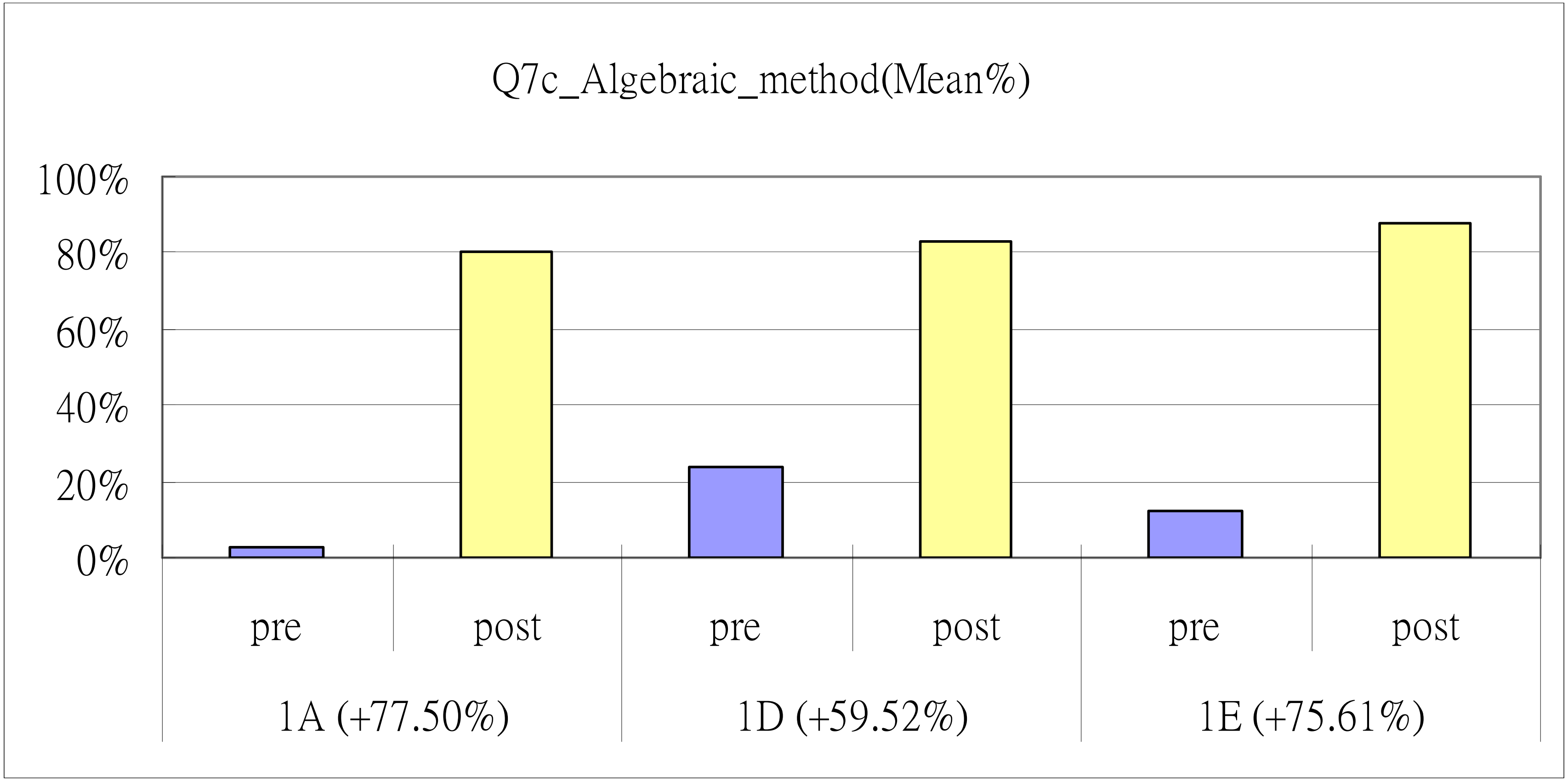
7. Given that $3b = 8c$, find $b : c$ by using the following methods:
(b) Substitution



Comparison of the pre- & post-test results

7. Given that $3b = 8c$, find $b : c$ by using the following methods:

(c) Algebraic



Comparison of the pre- & post-test results

7. Given that $3b = 8c$, find $b : c$ by using the following methods:

(d) Your own method, if any.

Try some integer

$\because 3b = 7c$

$\therefore 3(7) = 7(3)$

$21 = 21$

$\therefore b : c = 7 : 3$

$3b = 7c$

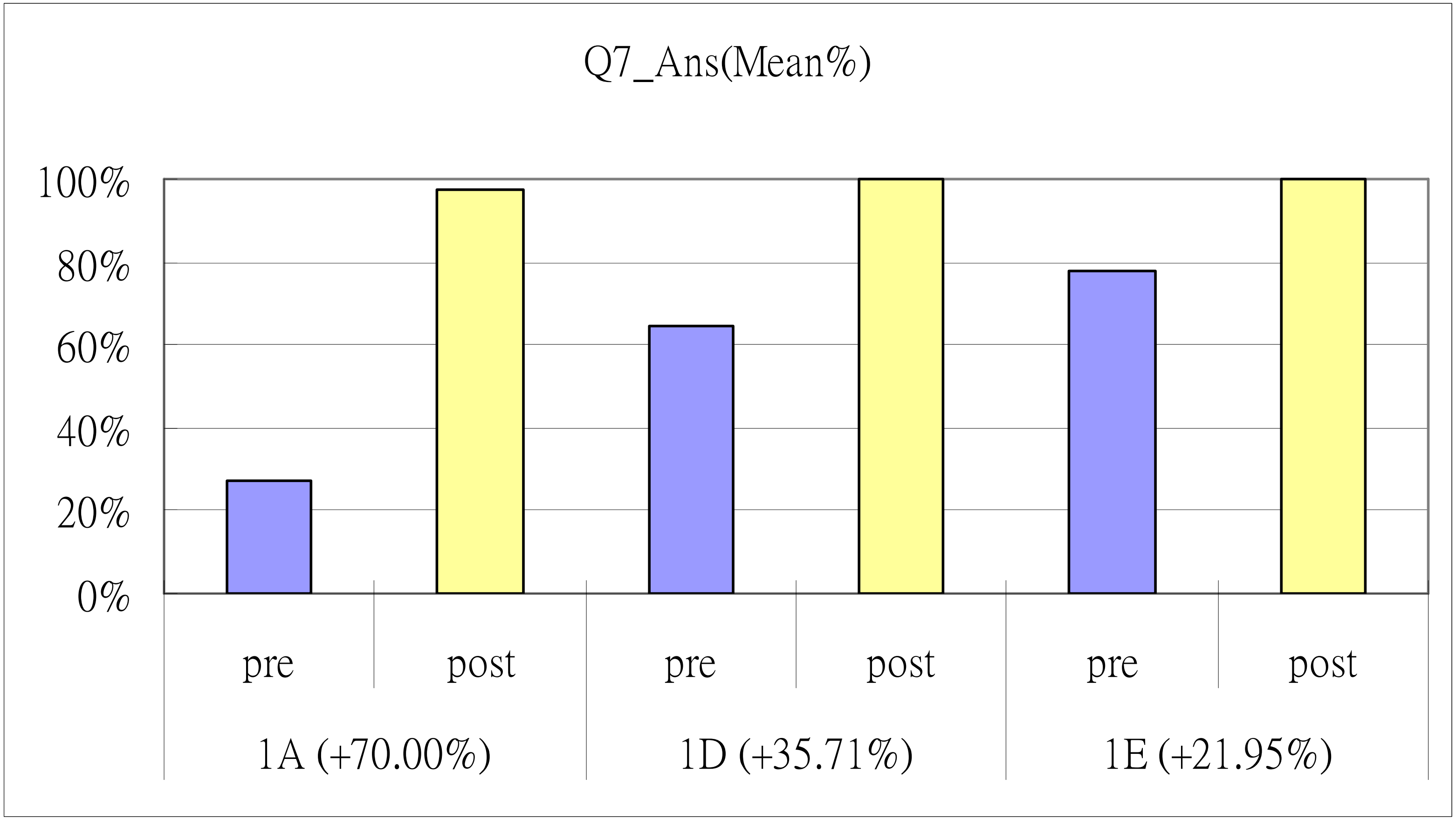
$3 \times 1 = 3$	$7 \times 1 = 7$
$3 \times 9 = 27$	$7 \times 2 = 14$
$3 \times 8 = 24$	$7 \times 3 = 21$
$3 \times 7 = 21$	$7 \times 4 = 28$

$\therefore b : c = 7 : 3$

Comparison of the pre- & post-test results

7. Given that $3b = 8c$, find $b : c$ by using the following methods:

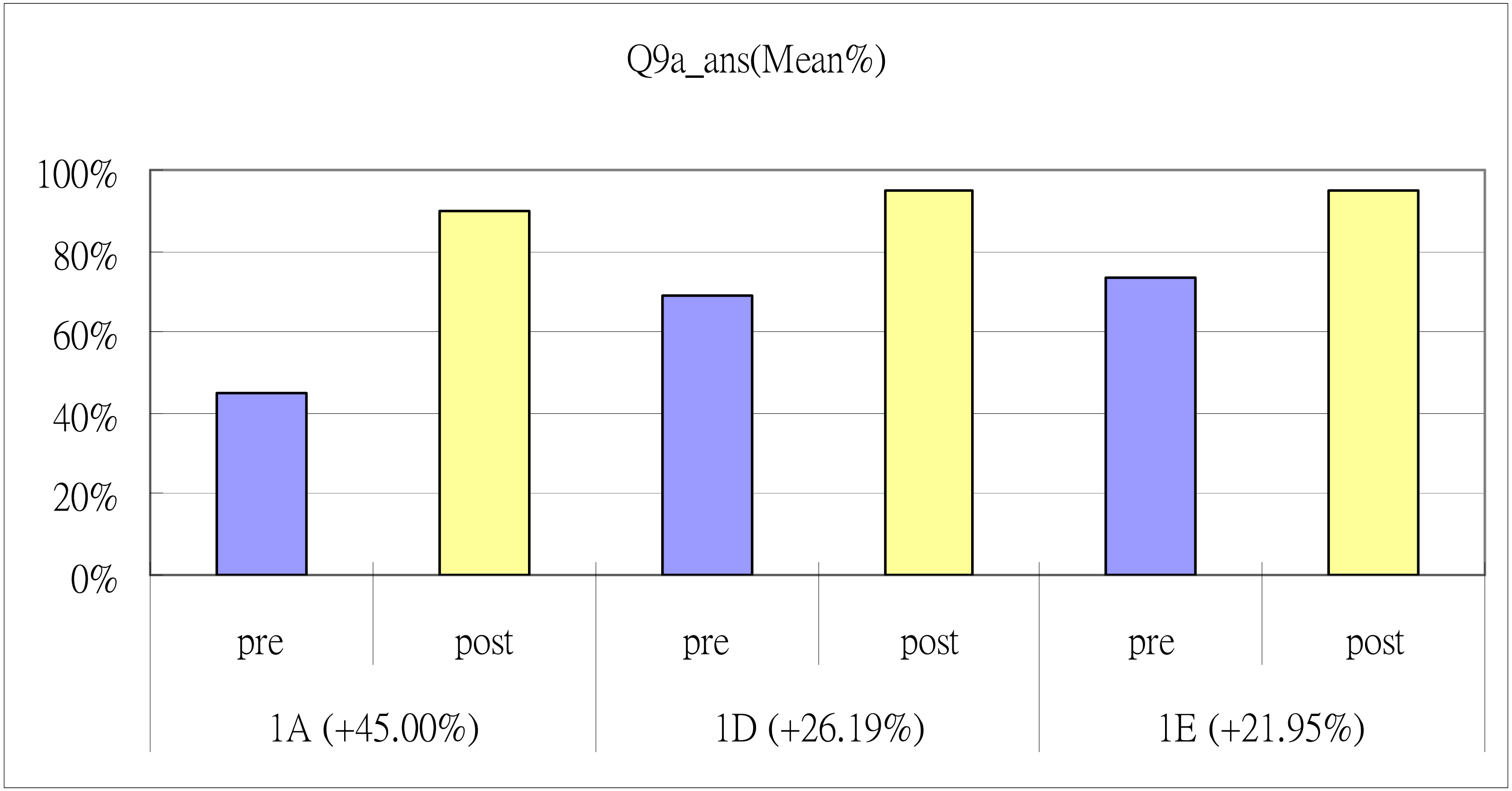
(a) L.C.M. (b) Substitution (c) Algebraic (d) Your own method, if any.



Comparison of the pre- & post-test results

9. Given that $6a = 7b = 9c$, find

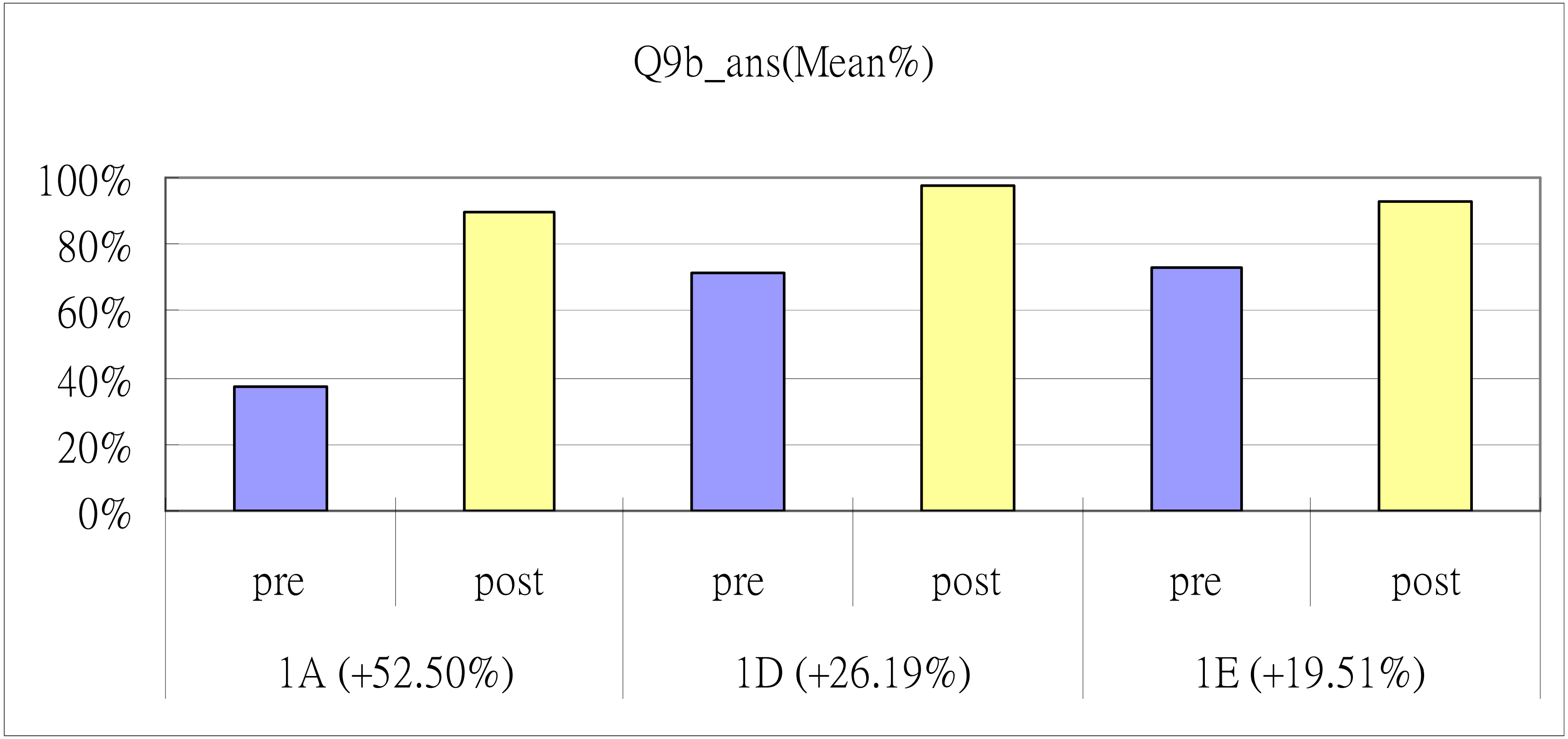
(a) $a : b$ (b) $b : c$ (c) $c : a$



Comparison of the pre- & post-test results

9. Given that 6a = 7b = 9c, find

(a) $a : b$ (b) $b : c$ (c) $c : a$



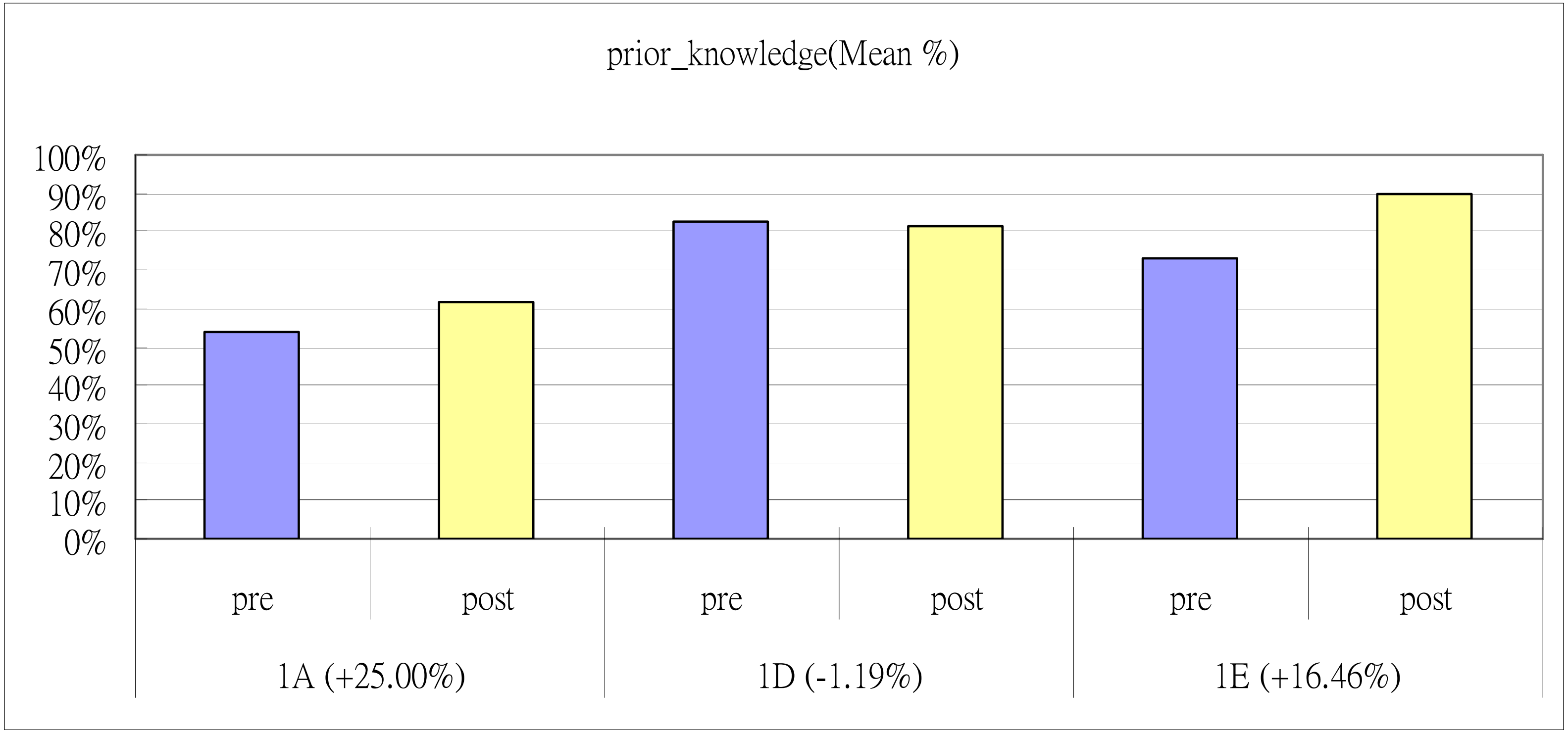
Comparison of the pre- & post-test results

Prior -knowledge

- understand that ratio form and fraction form can be used interchangeably

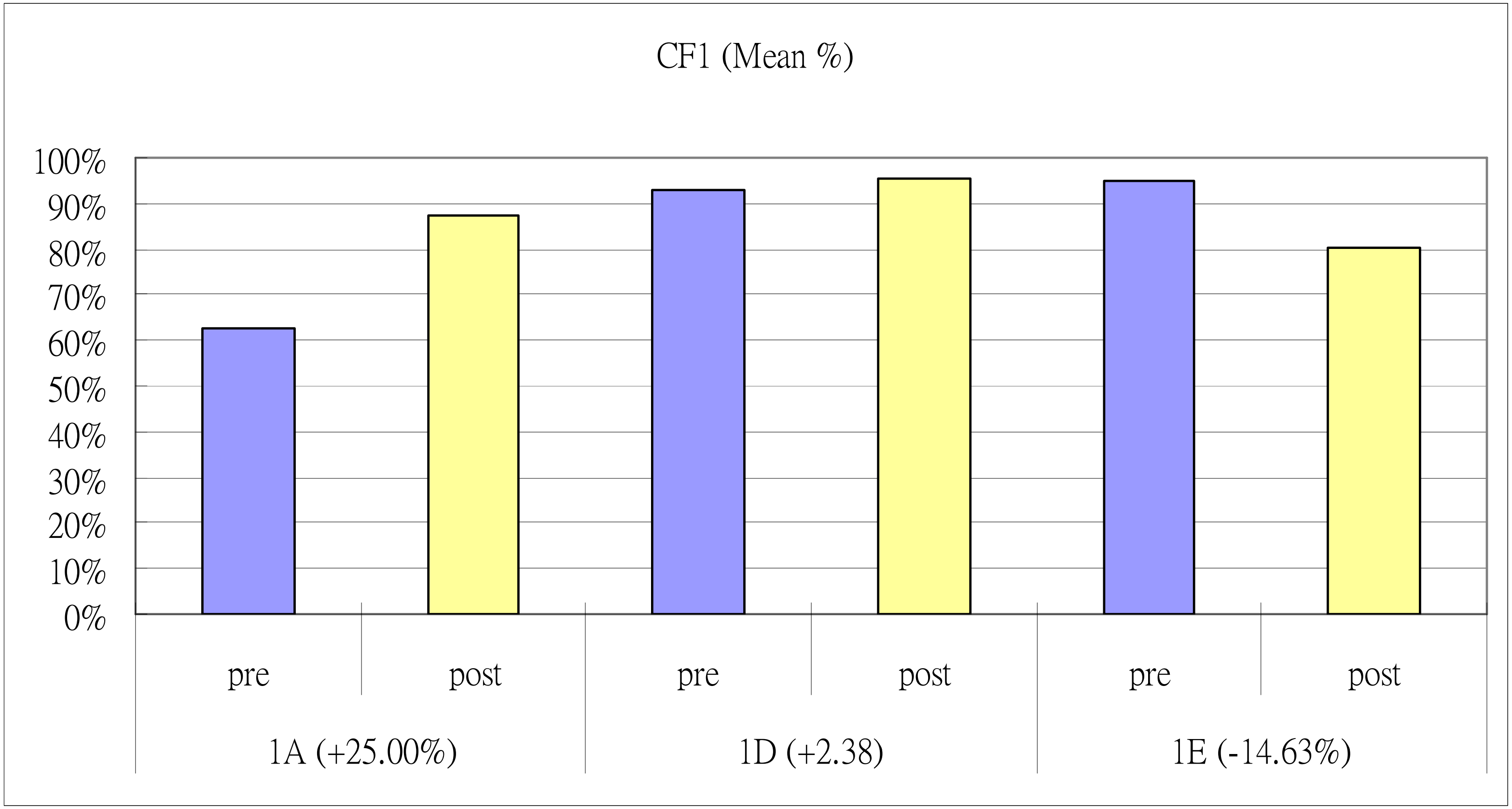
  a : b = $\frac{a}{b}$

- understand that two quantities in ratio form is based on the common unit.
- understand that the ratio doesn’t change when the components are multiplied by the same factor.



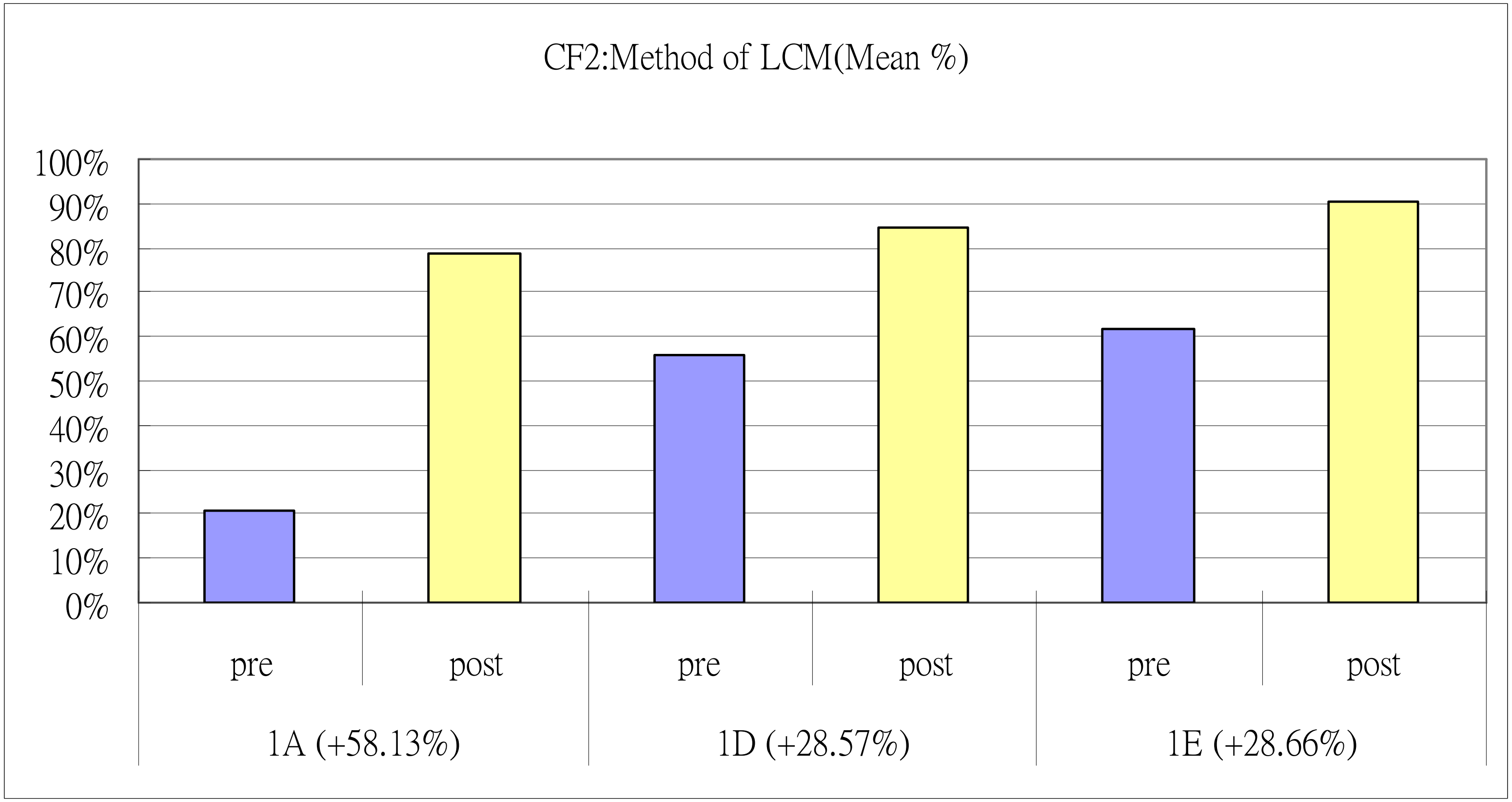
Comparison of the pre- & post-test results

CF1: To identify the larger quantity



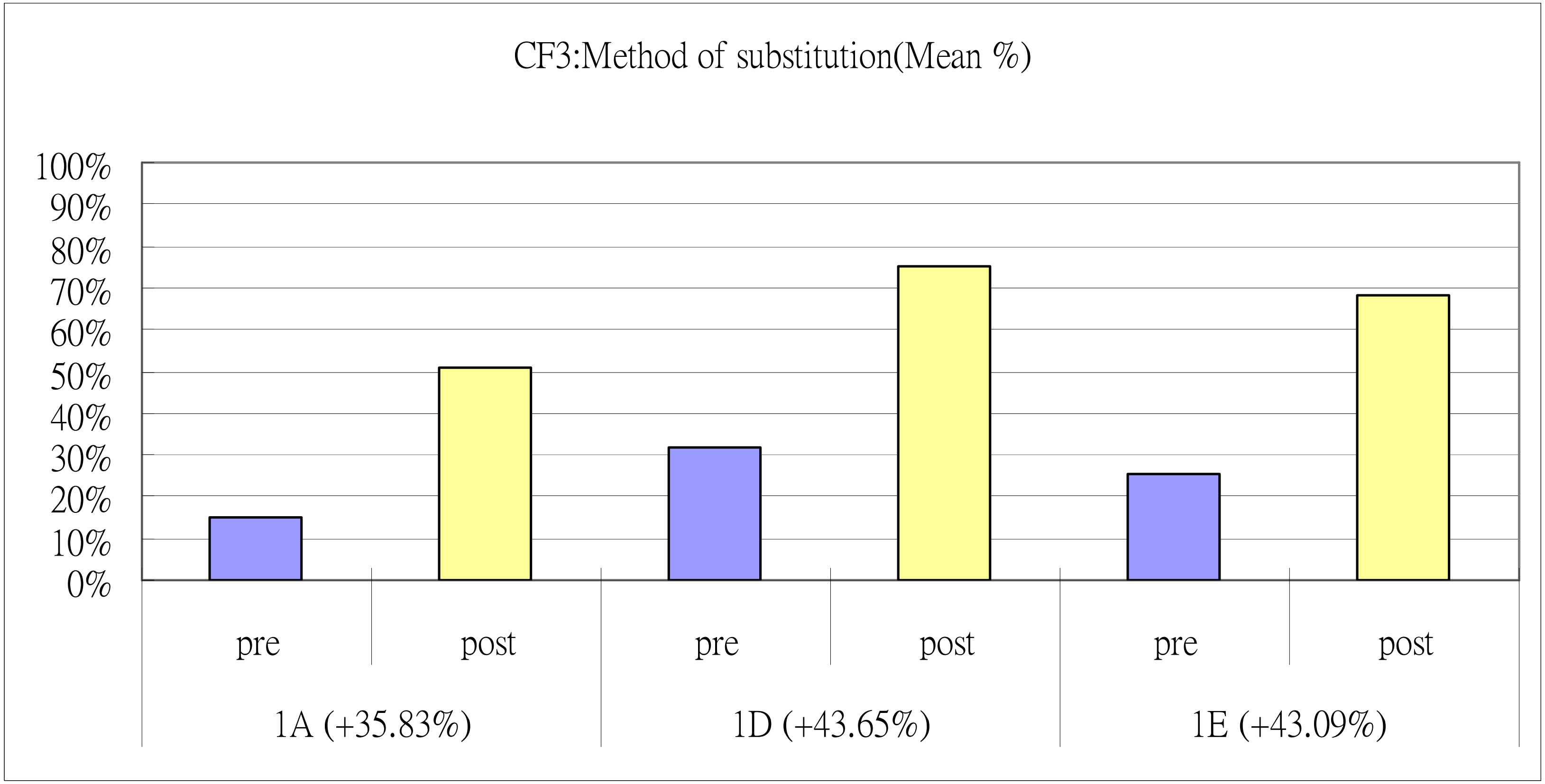
Comparison of the pre- & post-test results

CF2: Technique to find the ratio by LCM



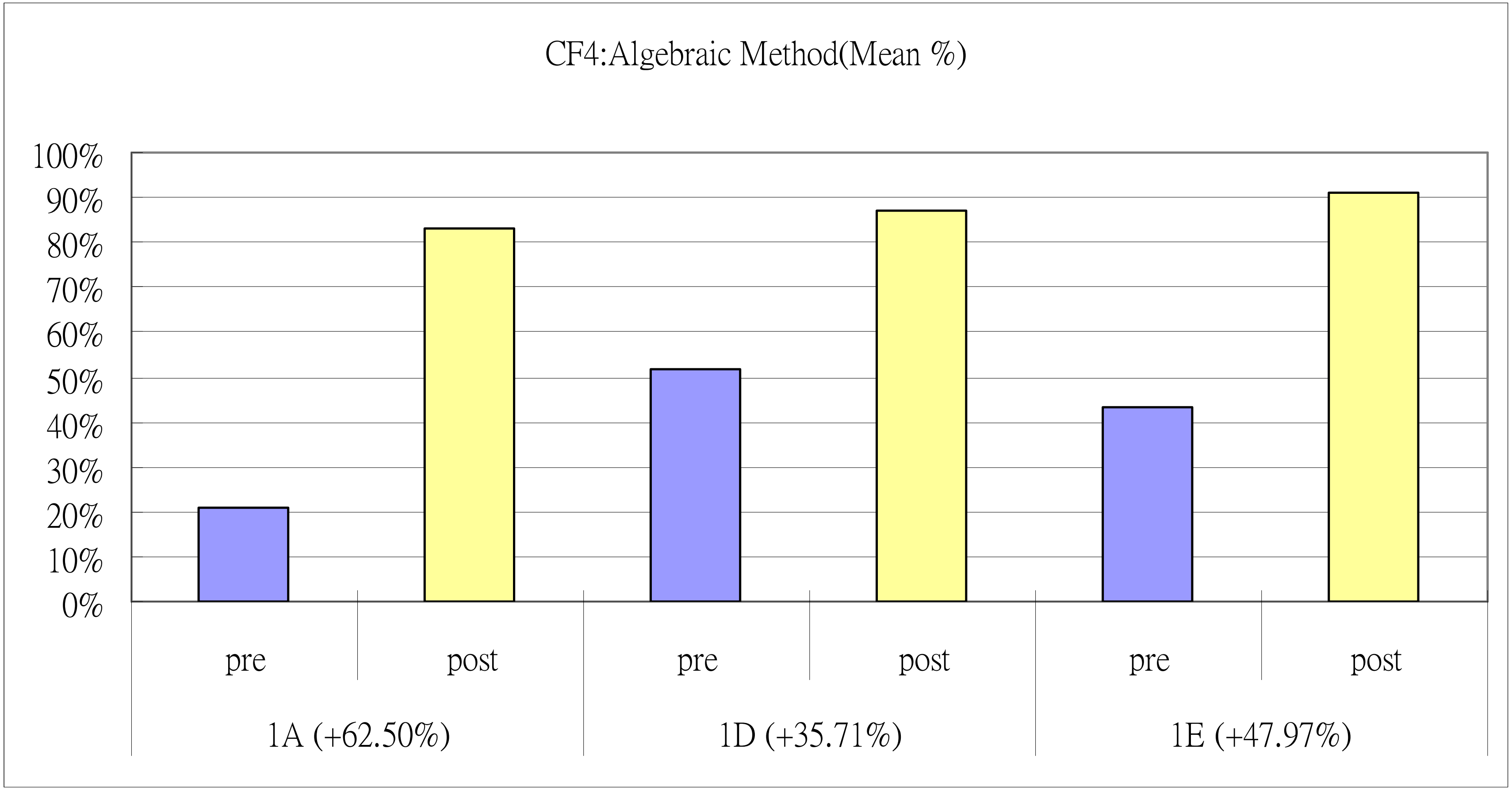
Comparison of the pre- & post-test results

CF3 : Technique to find the ratio by substitution



Comparison of the pre- & post-test results

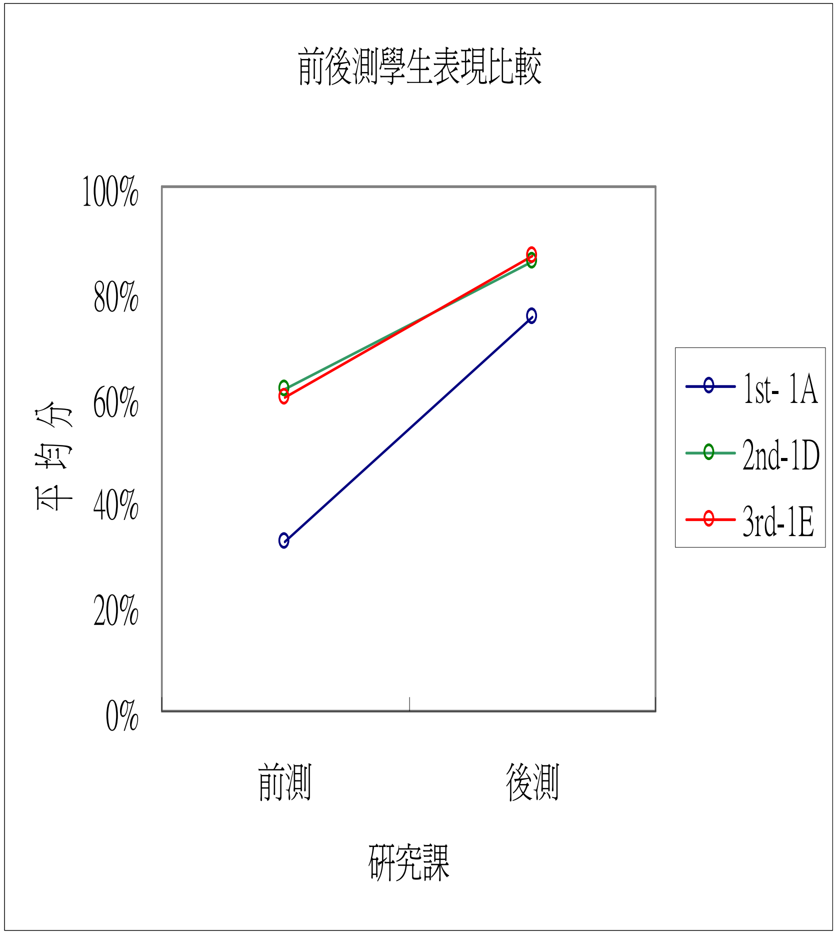
CF4 : Technique to find the ratio by simplify the equation into fraction form



Comparison of the pre- & post-test results

Overall Performance (By Teaching Cycles)

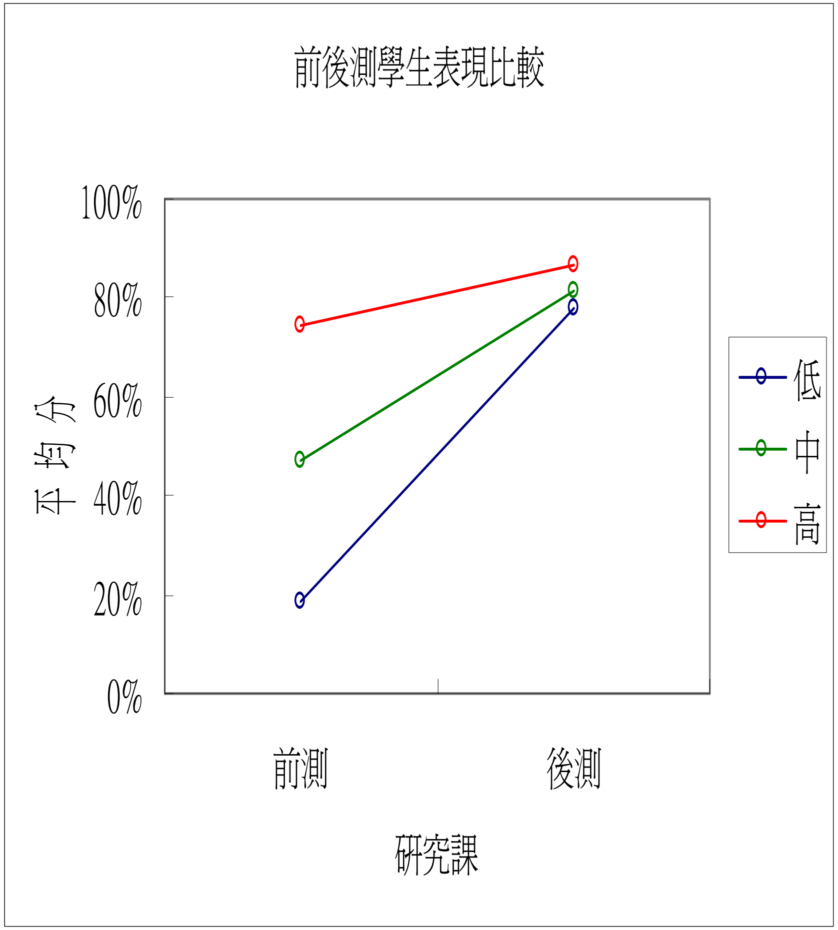
Teaching Cycle	No. of students	Pre-test (average)	Post-test (average)	effective
		%	%	%
1st – 1A	40	32.06 (18.21)	75.00 (13.37)	+42.94
2nd – 1D	42	61.40 (21.07)	85.52 (9.48)	+24.12
3rd – 1E	41	59.53 (20.65)	86.91 (11.48)	+27.38
合共	123	51.24 (23.96)	82.56 (12.60)	+31.32



Comparison of the pre- & post-test results

Overall Performance (By Performance of Pre-test)

Group by Performance of pre-test	No. of student	Average (pre-test)	Average (post-test)	effective	Group by Performance of pre-test
	%		%	%	%
High	72.01-100	52	74.13 (8.32)	86.58 (10.99)	+12.45
Middle	27.01–72	39	47.37 (9.81)	81.24 (11.69)	+33.87
Low	0 – 27	32	18.75 (6.40)	77.63 (14.27)	+58.88
Total	0 - 100	123	51.24 (23.96)	82.56 (12.60)	+31.32



Teachers’ reflection

- Without this lesson study, would have taught this topic according to the textbook just like past years
- Have a better understanding of student learning
- Will predict students’ learning difficulties in some challenging topics
- Will use various approaches to aspire students
- Have learned from each others
- Have felt great satisfaction and professional growth