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On using Geometer's Sketchpad to teach relative velocity

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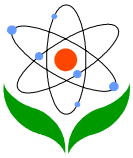**Singapore**

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Introduction

In the current Additional Mathematics syllabus (Ministry of Education, 2000), the topic Relative Velocity has been introduced. This topic involves abstract Mechanics concepts, which are difficult for both students and teachers. Many junior teachers might not have studied an undergraduate Mechanics course in their undergraduate days.

As Leong & Lim (2003) found out, effective use of Geometer's Sketchpad[*] (GSP) was

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[*]Geometer's Sketchpad - this is a useful tool in teaching concepts of geometry to Secondary School students. It has an official website which is located at http://www.keypress.com/sketchpad/.

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able to improve students' spatial ability measured by the students' performance in geometry achievement test. Since Mechanics, and Relative Velocity in particular, are built up from geometry concept, the use of GSP may be able to offer some hope in assisting students to visualize some concepts for this topic.

In this note, the uses of GSP to enhance students' understanding of some rudimentary kinematics concepts will be discussed. It is hoped that teachers will be able to illustrate the concepts without getting students into the rigors of tedious mathematical computation. All the complicated mathematical computation could be left to post-secondary level when the students take up a course in Applied Mathematics.

Computer software and learning of Geometry

Some senior teachers groan over the fact that the rigor of geometric proofs, which is very useful to train the students in logical thinking, is no longer in the current curriculum. The introduction of computer software into the curriculum always invites fear among the teachers of the loss of students' basic mathematical ability. For example, with the introduction of calculators, some teachers complained that students might lose their sense of estimates for basic operations on simple numbers. The same reasoning extends to the use of computer in education: with the use of GSP, would the rigor of training in logical training and the students' ability to solve geometry problems be affected?
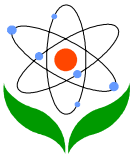
Hoehn (1997) gave sample activities of exploring worksheets that could be done with GSP. He suggested that activities that involve the proof of theorems on geometry could be done with the software. It is implied that more abstract results on geometry can be taught by focusing on improving students' spatial ability. Hoehn further suggested that students could be asked to attempt to generalize existing theorems and state and verify their conjectures. All this is in line with our new Problem-Solving Approach in the syllabus (Ministry of Education, 2000).

As Leong & Lim (2003) pointed out, the key features that make the software GSP suitable for teaching transformation geometry are
   (a) it enables objects to be transformed on the screen;
   (b) it allows easy measurement of distances, angles and areas;
   (c) it has the click-and-drag feature that enables users to experiment different cases
   (d) it allows animations of motion to be done.
Here the above features will be made use of to enable the teaching of concepts of mechanics more visual than rigorous proofs.

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Example 1: Crossing the river

For most students learning Relative Velocity, one of the main difficulties is the resolution of vectors. Thus it is important to aid students in visualizing projection of vectors before plunging into the actual concepts of Relative Velocity. Here GSP can be useful to assist students in visualizing concept of projection of vectors.

One main setting of Relative Velocity is the scenario on "crossing the river". We shall assume that the banks of the river are two parallel lines, as shown in the Figure 1 below. The distance between the two parallel banks, as represented by the two parallel lines, is d unit. We shall further assume that the true speed of the boat is kept constant at v at an angle q to the bank
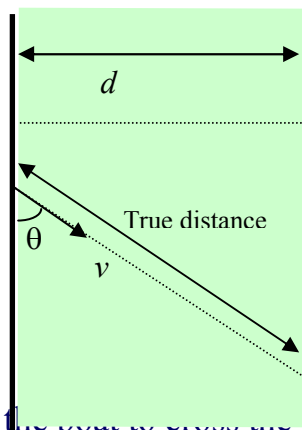
Then the time taken by the boat to cross the river

Figure 1

$$= \frac{\text{true distance}}{\text{true speed}} = \frac{\text{true distance}}{v} \qquad .....(A)$$

By considering the projection along the direction marked by the distance d in Figure 1 above, another way to compute the time taken to cross the river

$$= \frac{d}{v \sin \theta} . \qquad .....(B)$$

Both ways of computation, (A) and (B), give the same time for the boat to cross the river. It may not be easy to convince Secondary school students that (A) and (B) give the same time, since the concept of "projection" is not in their Secondary School Science curriculum.

For a student who is not particularly well versed in trigonometry and geometry, GSP can be used to illustrate this clearly. The worksheet using GSP is introduced in Figure 2 below.
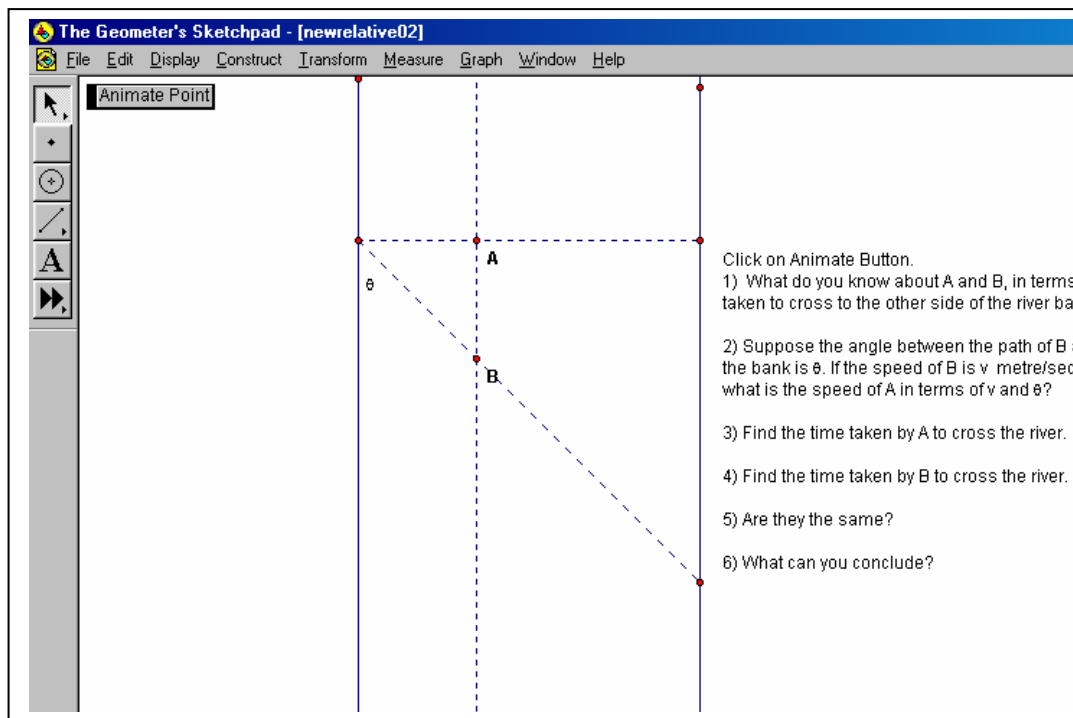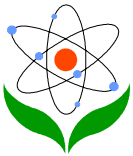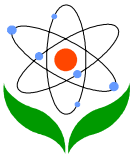
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Figure 2: Worksheet for Example 1

In the above worksheet, the students click the button Animate Point on the screen to start the animation of the two points A and B. Initially both points A and B coincide with the point X. Upon clicking the Animate Point button, both points start to move from left to right as shown in the above diagram.

The point B represents the actual boat crossing the river while the point A is another particle that is dependent on the movement of B such that (i) AB is always parallel to the bank; and (ii) A and B reach the other shore at the same time.

The time taken by the boat B to cross the river can be calculated as $\dfrac{\text{true distance}}{v}$, where the true distance refers to the actual distance traveled by the boat B. The time taken by the particle A, through the series of prompting questions in the worksheet (see Figure 2), can be computed as $\dfrac{d}{v \sin \theta}$. Since both particles A and B reach the other shore at the same time, another way of computing the time taken to reach the other shore can be taken as $\dfrac{d}{v \sin \theta}$. In many of the numerical questions in the typical examination questions, the use of $\dfrac{d}{v \sin \theta}$ to compute the time taken to cross the river may be much more

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straightforward.

Example 2: To reach the other shore in the quickest possible time

Another problem that can be clearly illustrated by using GSP is to find the shortest time taken to cross the river. Suppose a current flows at a certain velocity w downstream. A boat is steered at a constant speed v. The direction the course taken is to be determined by the boat. The question is: what is the direction the boat should steer in order to reach the other side of the bank in the shortest possible time?
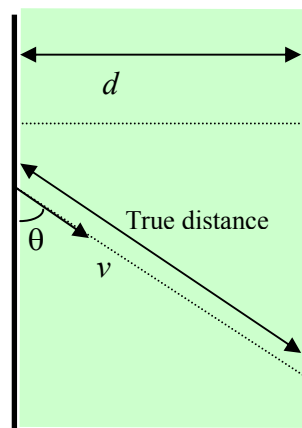

Figure 3

We construct the worksheet as in Figure 4 below. In Figures 4 and 5 below, the unit of measurement of time is in second.

The students are asked to drag the arrow, the direction of which represents the direction the boat is steered. Once decided on the direction the boat be steered, the button **ANIMATE POINT** on the screen can be clicked to demonstrate the path the actual boat travels (see Figure 5). The direction the boat travels represented by GE in Figure 5 will not be the same as the direction of steer of the boat, represented by GD. The point E represents the actual position of the boat while D is the position of the boat if there is no current downstream. Both points D and E have the same projection along the direction perpendicular to the two parallel river banks.

Students click and drag the course taken by the boat to allow for different choices of directions steered by the boat. By altering the course to be taken by the boat, the actual path traveled by the boat will be altered accordingly. This is immediately visible from the worksheet itself. The students can then be asked to explore on the course taken by the boat in order to reach the other shore in the quickest possible time by taking different courses in still water.
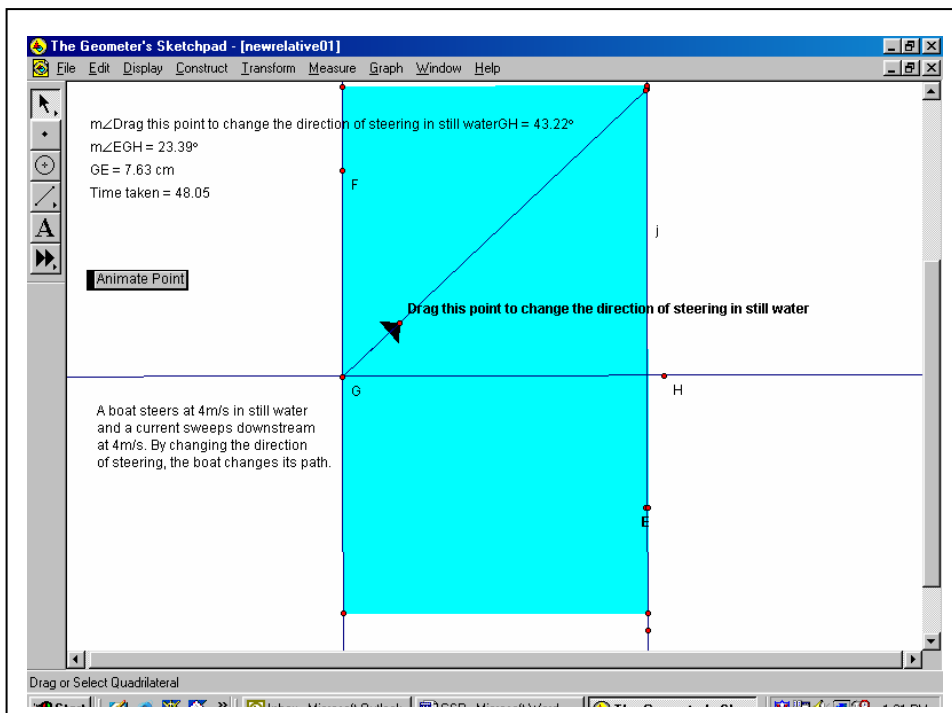
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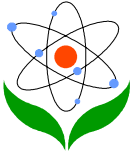**Tin-Lam TOH**
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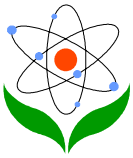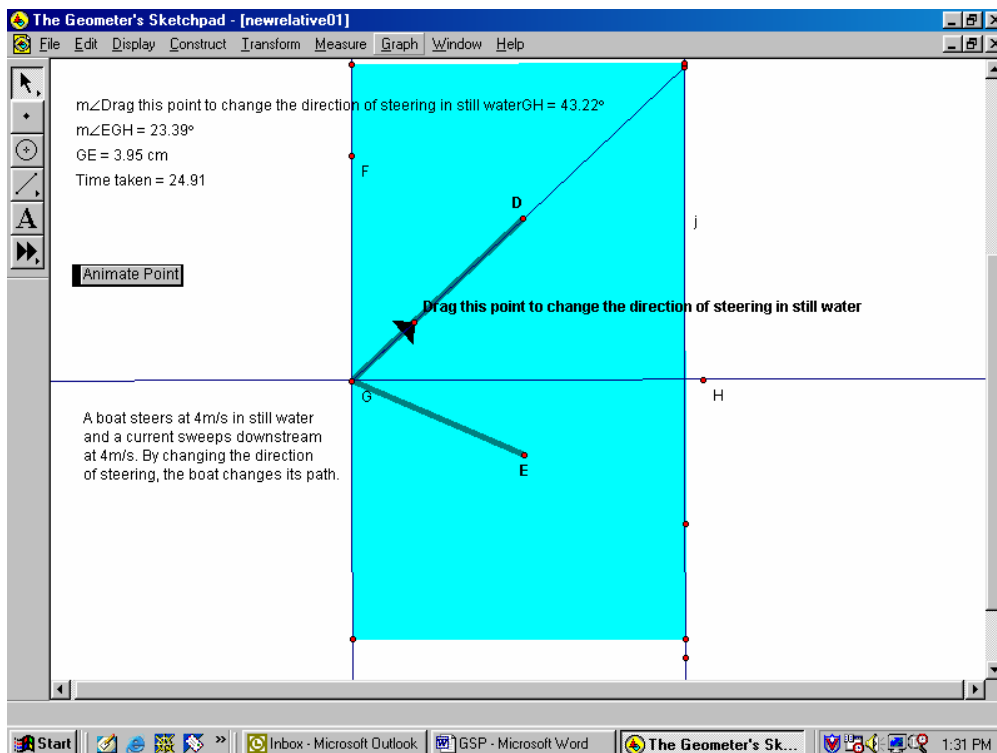
Figure 4

Figure 5

The students will discover that the course that the boat should take in order to reach the

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other shore in the quickest possible time is that perpendicular to the two parallel banks. The mathematical rigors of projection will be dispensed with while the objective is met.

As a bonus to the use of GSP in this case, the students will be able to make the following observations with the above file:

a. If the boatman steers upstream, the speed of the boat is slowed down compared with the speed he steers, and hence it takes a longer time to reach the other shore;

b. If the boatman steers downstream, the speed of the boat is increased from the speed he steers the boat (the further downstream, the faster the speed of the boat). However, the distance it has to take to cross the river is much longer, hence the time taken to reach the other shore is also increased.

Observations (a) and (b) above are difficult to demonstrate with the mechanical computation itself; with the use of such animation, the observations are quite visible to the students.

Example 3: Concept of the *actual system* and *relative system*

The method of teaching Relative Velocity is usually the "Reduction to Rest" method, see National Institute of Education 2003, Pp 186-189. In a system when two particles A and B are moving with constant velocity, it is equivalent to another system where one of the particle (say B) is resting while A is traveling with the relative velocity of A with respect to B. The former system is called the "Actual System" while the latter system the "Relative System with respect to B".

The question which naturally arises is: why are these two systems equivalent? From the geometry point of view, it is just a translation of the origin to the point A, which is supposed to be reduced to rest. However, this is abstract for most students as it implicitly involves translation of the system. With the use of animation, the concept of equivalence can be easily brought out.

Figure 6 shows the case when both the Actual System and the Relative System (B reduced to rest) are shown in parallel. As the animation button is clicked, both systems will start to move (as in Figure 7). In the Actual system, both particles A and B move whereas in the Relative Velocity, only particle A moves while B is at rest.

Students will observe that at any point in time, the vector AB, the relative displacement of A from B, in either the Actual System and the Relative System are equal.
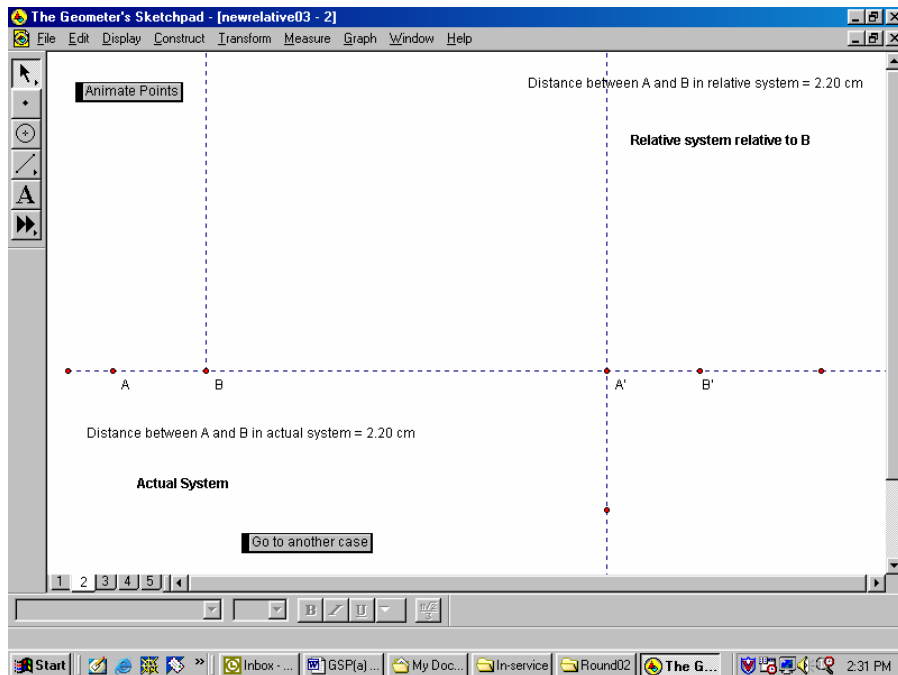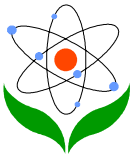
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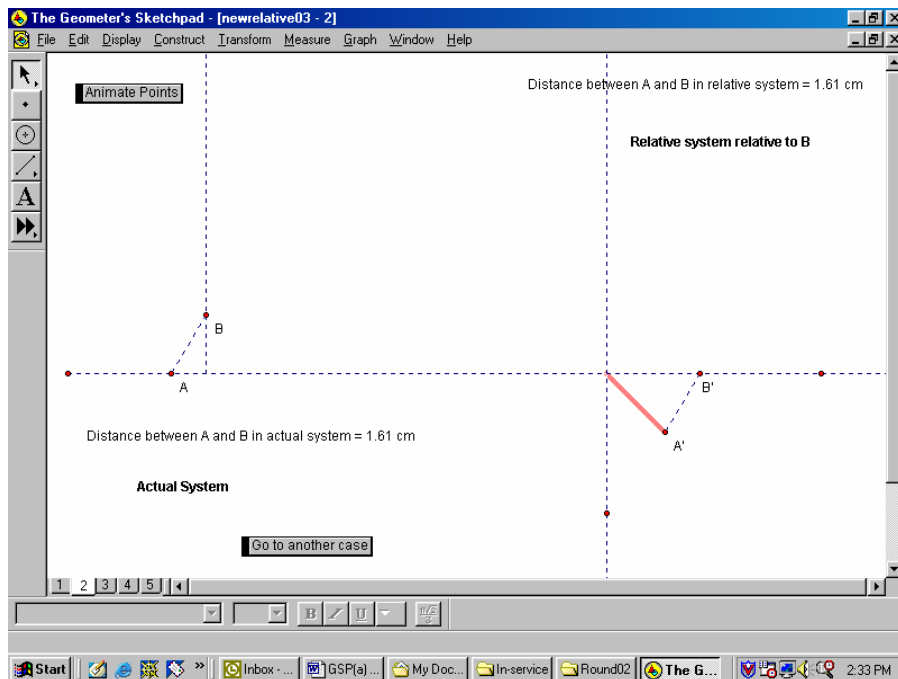**On using Geometer's Sketchpad to teach relative velocity**

Figure 6

Figure 7

Students can change the initial points of A and B by dragging them to different positions, but the vectors AB in both the Actual System and Relative System will always be equal, regardless of whether A and B are in motion or stationary. The physical meaning of the Relative System is clearly brought out. This is an alternative approach to mechanically "converting" the Actual System into the Relative System by reducing B to rest.

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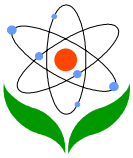
Teachers' content knowledge

In order to be able to design worksheets suitable for their students, teachers need to have sound pedagogical content knowledge - the amount of knowledge required in the Additional mathematics syllabus alone is not sufficient. Usiskin (2001) classifies the *pedagogical content knowledge* that a teacher needs to have under three categories: (1) generalized knowledge from what is required in the syllabus; (2) concept analysis and (3) problem analysis. Possibly one of the reasons why current in-service teachers find this topic difficult is that they do not have sufficient pedagogical content knowledge for this topic. As such, they find it difficult to teach this topic or to design suitable activities to facilitate their students' learning.
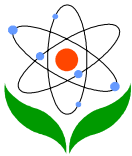
Value added by using GSP

Consider Example 1: students could acquire a sound understanding of the concept of projection of a vector (in this case velocity vector) along a given direction (the direction that is perpendicular to the two parallel river banks) by visual impact, rather than by the traditional rote memory of equating the two expressions $\frac{\text{true distance}}{\text{true speed}} = \frac{d}{v \sin \theta}$ (see Example 1). Resolution of vectors forms the foundation of Relative Velocity. To be confident in designing worksheets pertaining to this area, teachers need to be able to see this from the geometrical concept of similar figures and the related mechanics concept of resolving vectors.

Example 2 is another difficult concept for students to visualize: irregardless of the water speed of the boat, the boat must be steered perpendicular to the two parallel banks in order to reach the other shore in the shortest possible time. However, with the use of GSP, the following facts can be easily visualized by dragging the different directions of the boat: if the boat is steered upstream, it takes a longer time to reach the other shore since the true speed of the boat is reduced; if the boat is steered downstream, even though the speed of the boat is increased, it has to travel a much longer distance in order to reach the other shore. Thus, in order to reach the other shore in the shortest possible time, he must steer perpendicular to the two parallel banks.

Example 3 needs the physical interpretation of the subtraction of two vectors AB as 'treating' A as the origin. So, it is geometrically equivalent to translating the point A back to the same point at all time. The vector AB is the same in both systems. This was the relation of the Relative System and the Actual System. Thus, the concept of "Reduction to Rest" method of solving problems involving Relative Velocity can be given a more meaningful physical interpretation by using GSP.

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Feedback on using GSP to teach *relative velocity*

The author has conducted a 12-hour workshop on Teaching Relative Velocity for in-service teachers in Singapore. A total of 2-hour was spent on using GSP, with hands-on practice for teachers with the use of appropriate worksheets. At the end of the entire course, participants' feedback was collected. Two questions pertaining the use of GSP to teach the above concepts related to Relative Velocity were asked and the feedback were as follows:

Questionnaire

	Strongly Disagree	Disagree	Agree	Strongly Agree	Not Applicable
I think the use of GSP is useful to illustrate the related Mechanics concepts	0	1	16	18	-
I would want to try out using GSP to teach Mechanics	0	3	13	18	1

Out of the 35 participants, 18 of them (more than 50%) regarded the GSP section of the workshop as being the most useful of the entire course.
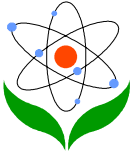
Currently, the author is working with two schoolteachers who will be teaching Relative Velocity in 2004 in order to obtain any effect on the students' performance after attending lessons on Relative Velocity using GSP.

Conclusion

Teachers can design discovery worksheets to allow students to discover and verify the necessary and sufficient conditions for two particles moving with constant velocities and with different starting points to intercept. All these activities enable students to make sense of all the standard results of Relative Velocity without having to get into the rigors of mathematical computation.

The above discussion gives three examples on how GSP can be used to facilitate students to visualize some key concepts in the topic of Relative Velocity. It should be noted that the above are some recommendations on how the software can be used for teaching; it does not exhaust all possibilities on how the software can be used in teaching the topic. The rest depends on the teacher's creativity in designing their files and related worksheet for their students.

Educators should not overlook what matters most in ensuring the effective use of technology in classrooms (Bliss T.J., Bliss L.L., 2003). Besides having a sound pedagogical content knowledge, teachers must be equipped with reasonable level of

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proficiency of the tool itself. In addition, teachers should constantly bear in mind that the use of GSP or any other educational technology is to enhance students' learning.

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