Evaluation of teacher candidates' knowledge about vectors

Hanife SARAÇOĞLU* and Özge KOL

Ondokuz Mayıs University, Faculty of Education, Department of Mathematics and Science Education, Samsun, TURKEY

*** Corresponding Author's E-mail: hanifesa@omu.edu.tr**

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Contents

Abstract

In this study, it is aimed to reveal math teachers' knowledge level and misconceptions about vectors. The study was conducted with the participation of 66 teacher candidates in different nine universities. As for data collection tool, a questionnaire of 6 open-ended questions was used, related to basic knowledge about vector magnitude, addition, subtraction, scalar and vector products. In conclusion, we found that math teacher candidates had difficulty with notation for the magnitude of vector and did not interpret the direction of vector product. We observed that the female mathematics teacher candidates showed more successfully than male teacher candidates in vector properties.

Keywords: Physics Education, Vectors, Transfer, Vector Product, Scalar Product.

Introduction

Basic science concepts have been considered as prerequisite for the understanding and explanation of subsequent science topics related to these concepts and they also take the responsibility for making sense of the associated concepts (Mann and Treagust, 2010). The some subjects of physics and mathematics also overlap and enrich one another with complementary perspectives. One of them is also vectors. Vectors are essential component of the mathematic language of the physics, even at the introductory level (Knight, 1995). Students require a good grasp of basic vector concepts to succeed in a physics course (Sheets, 1998).

The concept of vector can be associated to almost any topic in physics, but the shortcomings in the process of learning can lead to serious problems (Aguirre, 1988; Aguirre and Rankin, 1989; Barniol and Zavala, 2015; Zavala and Barniol, 2013). Especially while adding, subtracting, and identifying unit vector, identifying the magnitude as well as the direction of the vector tends to contribute towards the difficulty of the problem (Barniol and Zavala, 2010; Barniol and Zavala, 2012; Barniol and Zavala, 2014; D'Angelo, 2010; Flores, Kanim and Kautz, 2004; Hawkins, Thompson and Wittmann, 2009; Knight, 1995; Nguyen and Meltzer, 2003; Schaffer and McDermott, 2005; Van Deventer and Wittmann, 2007).

Redish (2005) with university physics students in classes from algebra-based introductory physics indicates that the gap between what students think they are supposed to be doing and what their instructors expect them to do can cause severe problems. Despite the fact that vector illustration is the easiest way for scientists of representing some concepts, it can be confusing or even inextricable for students. The most of students seem incapable of reasoning with vectors as abstract elements of a linear space (Hestenes, 2002). It can be that the learning problems of students in mathematics are transferred to the learning environment in physics (Basson, 2002). Mestre (2001) indicates from his own experience and from research findings that transfer is not easy to accomplish. As new knowledge is learned, students should be assisted in considering multiple contexts and in linking that knowledge to previously learned knowledge. The ways in which the students perceive the world in their past experiences influence the learning of the concept. It is known from the literature that students have some preconceptions from their experiences and a lot of them do not match with the scientific conceptions, as named misconceptions, alternative conceptions or alternative framework (Halloun and Hestenes, 1985; McDermott, 1984). Misconceptions are difficult to change and may affect how learners process new information and data (Beydoğan, 1998; Gilbert, Osborne and Fensham, 1982; Helm and Novak, 1983; Watts and Pope, 1989). It can be observed from studies conducted related to physics education that students have many misconceptions while learning concepts about vectors (Aguirre and Erickson, 1984; Flores et al. 2004; Heckler and Scaife, 2015; Knight, 1995; Nguyen & Meltzer, 2003; Schaffer & McDermott, 2005). Hence these misconceptions should be diagnosed and teaching should be designed to take students' conceptions into account (Dekkers and Thijs, 1998; Duit and Treagust, 1995; Hewson and Hewson 1984; Osborne and Wittrock, 1983).

Some researches carried out studies about how vectors scalar and vector product is performed (Knight, 1995; Van Deventer, 2008; Van Deventer & Wittmann, 2007; Zavala and Barniol, 2010) and its geometric interpretation (Van Deventer, 2008; Zavala and Barniol, 2010). While Van Deventer prepared multiple-choice questions regarding scalar structure of scalar product's results, other scientists conducted researches on identifying challenges faced by students as they tried to differentiate torque from force as well as torque magnitude (Ortiz, Heron and Shaffer, 2005; Rimoldini and Singh, 2005; Van Deventer, 2008).

This study aims to determine the level of knowledge and misconceptions of mathematics teacher-candidates with open-ended questions about vector properties and operation. The questions we hope to answer with this investigation are: 1) whether it is written the correct representation of vector magnitude and vector; 2) whether it can be determined the direction of a third vector is perpendicular to the plane that contains two vectors multiplied vector product by using the right-hand side; 3) whether mathematics teacher-candidates correctly distinguish the cosine or sine of the angle in dot or vector product of two vectors; 4) whether mathematics teacher-candidates achieve to transfer form algebra classes while the questions with scalar and vector product are answered.

Method

Research Model

Qualitative research model that used in this study can be preferred to reveal perceptions and events in a realistic and holistic manner in their natural environments. The reason behind preferring qualitative research method is to obtain profound data, observe the research topic from the perspective of the participants, and to reveal the structure and the processes that constitute these perspectives (Yıldırım and Şimşek, 2006; Miles and Huberman, 1994). The research methodology of the study is the case study. The focus of this case is to identify levels of knowledge and misconceptions of mathematics teacher candidates about vector properties and operation.

Workgroup

The workgroup consists of 66 math-teacher-candidates from different nine universities, in 2015-2016 summer semester. 50 of them were female and 16 were male. Math teacher candidates were enumerated as female (F) and male (M) considering the gender factor.

Data Collection Tools

A questionnaire that included 6 open-ended questions was used in the study, questions are available in the annex. First question in the questionnaire was obtained from the study by Nguyen & Meltzer (2003); whereas second and third questions from the study by Küçüközer (2009). The other three questions were developed originally by researchers. Following the preparation of the questionnaire, validity of the questions in terms of content and purpose of the research was checked by two experts in the fields of physics and mathematics.

Collection and Analysis of the Data

Findings of the research were obtained by an open-ended questionnaire. Questions were categorized on two sections as vector properties and operation. Student responses and remarks were encoded by researchers separately, then grouped unanimously to be analyzed as full understanding (FU), partial understanding (PU), misunderstanding (MU), and not understanding (NU) (Abraham, Williamsom and Wetsbrook, 1994).

Responses from the teacher-candidates were analyzed qualitatively based on content, and results of the analysis were supported by direct quotes from the student statements (Downe-Wamboldt, 1992; Krippendorff, 1980; Sandelowski, 1995). During quotation from student statements math-teacher candidates were numbered respectively starting from one, and taking the gender factor into consideration, each number was prefixed with letters (F) and (M), indicating 'female' and 'male'.

Results

Findings obtained from the questionnaire regarding vectors were grouped and interpreted under five themes: (1) the length of vector, (2) vector addition, (3) subtraction, (4) scalar and (5) vector product.

Theme 1: The length of vector

Physical quantities are categorized in two as those which only have magnitude with an appropriate unit, and those which have both magnitude and direction (Adams, Bogduk, Burton and Dolan, 2006). Mass, time, volume, temperature and energy can be considered as simple examples to those which only have magnitude. Vectors are a method of expression used for quantities which have direction in addition to magnitude. Displacement, velocity, acceleration, force and momentum are examples of vector quantities. It was investigated whether teacher candidates could correctly compare the magnitude of vector with the first question, 12.12% of all the math teacher candidates gave the exact correct answer; whereas 74.24% displayed partial answer. It is also more or less equal the percentages of partial answer for female and male teacher candidates. A teacher candidate [F_{20}] for example is in confusion about the magnitude of vectors and the vectors in terms of their illustration, stating: " **A**, **D**, **F**, **G** *vector magnitudes are equal*. **E**, **H**, **I** *vectors are equal*". Four vectors **A**, **D**, **F** and **G** can be defined to equal if they have the same magnitude and if they show the same direction. But, the magnitudes of vectors are equal only if they are same length, which have to represent |**A**| = |**D**| = |**F**| = |**G**|. The teacher candidate [F_{20}] knows the magnitudes of the vectors but does not know the representation of vector magnitude. It is different to representation of vector magnitude and vector. This information that should have been successfully processed through working memory is held in long-term memory (Carlson, Chandler and Sweller, 2003). But, when vector information was presented to [F_{20}], it could have not been successfully processed through working memory. When designing instruction, the mental load of [F_{20}] may be exceed limits of her working memory. Then, it can be said that she has an imperfect schema for vector. 13.64% of the students gave wrong answers to this question. Teacher candidate [F_8] coded with gave wrong answer as "**A**, **E**, **D**, **G**, **F** *magnitudes* *of vectors are equal*". A partial understanding example of the teacher candidate coded [M_6] response is given in Fig.1. All teacher candidates attempted to give an answer to this question; no blank, repetitive, or unrelated answers were encountered.

Figure 1. A partial understanding example of the teacher candidate response to the length of vector.

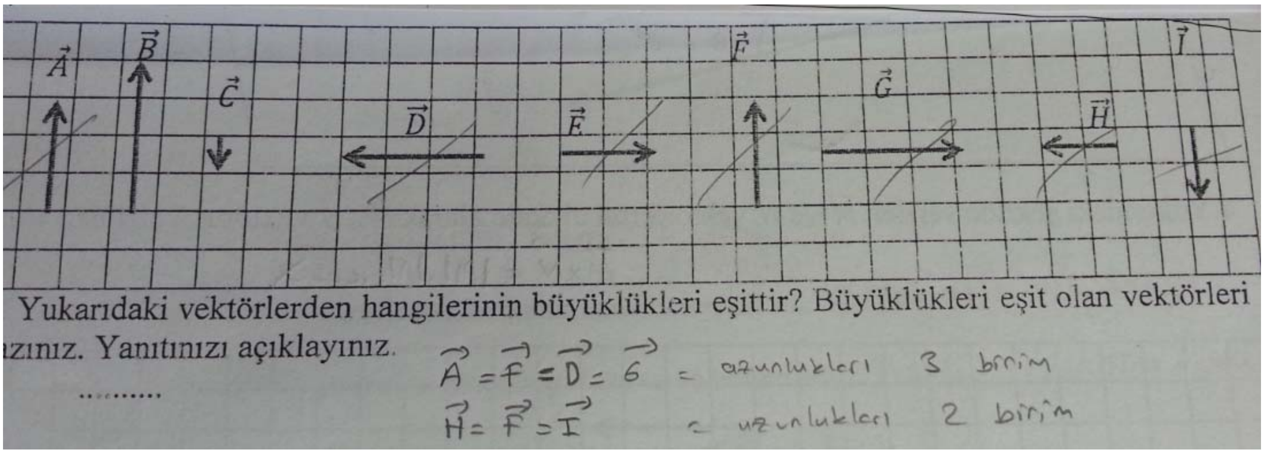

Table 1. Analysis of the question themed the length of vector

Comprehension Level	Encoding	f(%)	Female/ Male (%)
FU	. For vectors whose magnitudes are equal, it has to written $\|\vec{A}\|=\|\vec{D}\|=\|\vec{F}\|=\|\vec{G}\|$ and $\|\vec{E}\|=\|\vec{H}\|=\|\vec{I}\|$.	8 (12.12)	14 / 6
PU	. Vectors whose magnitudes are equal are known correctly, but it is used to wrong handwriting style.	49 (74.24)	74 / 75
MU	. Vectors which are same length, sense and direction are equal. . The magnitude of vector is defined on a line. . When vector is turned opposite direction, the sign of it changes.	9 (13.64)	12 / 19
NU	. Non-encodable or unanswered	-	-/-

Theme 2: Vector Addition

Vectors may be collected graphically or analytically (Halliday, Resnick and Walker, 1993). Both methods are being taught to each student participating in physics lectures on undergraduate levels, and it is always emphasized that it would be more practical to prefer analytical method as the number of vectors increase. In this study, a question is given about addition of two vectors using graphical method in two dimensions. Graphical methods for adding two vectors are known as the triangle and parallelogram rule of addition (Radi and Rasmussen, 2013).

While two vectors are adding, it is preferred with triangle law of vector addition by 51 math teacher candidates. 15.15% of the students were able to answer the question with partial understanding. Seven out of ten students answering the question with partial understanding added the vector with triangle law, but drew opposite direction for the resultant vector. The percentage of female math teacher candidates is bigger than the percentage of male math teacher candidates on full understanding level. All of the teacher-candidates were able to give an answer to the second question.

77.27% of the math-teacher candidates gave the correct answer to this question as showed Table 2.

Table 2. Analysis of the question themed vector addition

Comprehension Level	Encoding	f(%)	Female/ Male (%)
FU	. Two vectors are added by using the triangle method of addition. . Two vectors are added by using the parallelogram method of addition.	51 (77.27)	82 / 62
PU	. The resultant vector $\vec{R}$ is drawn from the head of the second vector to the tail of the first vector. . Two vectors are added, but the resultant vector is only drawn a line.	10 (15.15)	14 / 19
MU	. The tails of two vectors are superposed and then the resultant vector is drawn from the head of the first vector to the head of the second vector. . Not given two vectors are added. . The first vector and the negative of the second vector are added.	5 (7.58)	4 / 19
NU	. Non-encodable or unanswered	-	- / -

Theme 3: Vector Subtraction

In the process of vector subtraction, it is the method to utilize from the definition of the negative of a vector. The operation of **A - B** is defined as vector **-B** added to vector **A**.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

It is possible to take a look at vector subtraction from another perspective: The difference **A - B** of two vectors such as **A** and **B**, is a vector that needs to be added to the second vector in order to obtain the first. In this case the vector **A - B** is the vector drawn from the head of the second vector towards the tip of the first (Serway and Jevett, 2008).

Table 3. Analysis of the question themed vector subtraction

Comprehension Level	Encoding	f(%)	Female/ Male(%)
FU	. It is drawn the difference vector that is from the head of the $\vec{K}$ vector to the tip of the vector $\vec{R}$. . The vector $\vec{R}$ and vector $-\vec{K}$ are added, and the difference vector is drawn from the tail of the vector $\vec{R}$ to the head of the negative vector $\vec{K}$.	43 (65.15)	72 / 44
PU	. The vector $\vec{R}$ and vector $-\vec{K}$ are added, but the difference vector is drawn from the head of the vector $\vec{R}$ to the tail of the negative vector $\vec{K}$ . . The difference vector is drawn opposite direction. . The difference vector is drawn a line. . The negative vector $\vec{K}$ is named as the vector $\vec{K}$.	8 (12.12)	6 / 32
MU	. Two vectors are added instead of subtract. . The vector $\vec{K}$ and the vector $\vec{R}$ are combined head to head.	13 (19.70)	22 / 12
NU	. Non-encodable or unanswered	2 (3.03)	- / 12

It is seen that the problem was answered with a rate of 65.15% with full understanding (Table 3). In partial understanding level, the rate of female math teacher candidates is quite smaller than male math teacher candidates. Eight students performed subtraction with partial understanding. The most common conceptual delusion in vector subtraction is found the negative of the vector. While 19.70% of the teacher candidates provided wrong answers to the subtraction, 3.03% of them gave unaccountable answers.

Theme 4: Scalar Product

Scalar product of the vectors of **A** and **B** is expressed as **A.B**, this product is also known as dot product. A full understanding example of the teacher candidate coded [M_{16}] response is given in Fig. 2. It is defined as **A.B** = |**A**||**B**|cos(**A**, **B**). The scalar product of two vectors is a scalar quantity. The combination of |**A**||**B**|cos(**A**, **B**) occurs frequently in physics class. The geometric significance of the inner product **A.B** is also similiar from the standard vector scalar product (Hestenes, 2002).

Table 4 shows that only 6.06% of the teacher candidates answered the question with full understanding. With partial understanding, the question was answered with the rate of 22.73%. The teacher candidate numbered [M_7] was able to answer the question as: "**S.T > S.U > S.V** *due to the increase of angles between them*"; however, he confused the angle itself with the cosine of the angle determining the multiplication result in scalar product. Teacher-candidate numbered [M_{13}] answered the question as: "**S.T > S.U > SV** *Adding them tail of the first one to the tip of the last one, I chose the ones with larger cross vectors*"; displaying conceptual delusion about scalar product and vector addition.

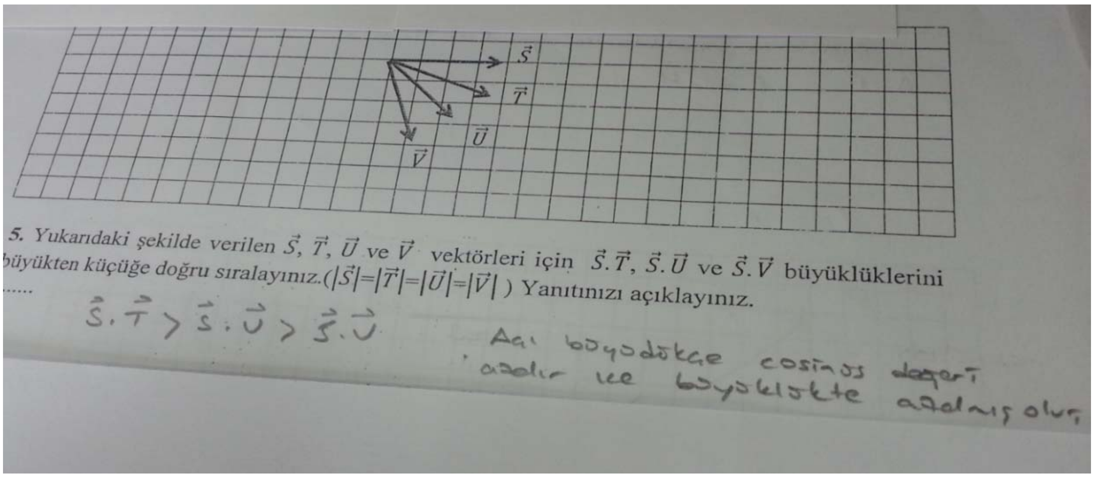

Figure 2. A full understanding example of the teacher candidate response to the scalar product.

Table 4. Analysis of the question themed scalar product of vectors

Comprehension Level	Encoding	f(%)	Female/ Male (%)
FU	$\vec{S}.\vec{T} > \vec{S}.\vec{U} > \vec{S}.\vec{V}$ is expressed, since $\vec{S}\ .\ \vec{T} = \lvert\vec{S}\rvert\lvert\vec{T}\rvert\cos(\vec{S}\ ,\ \vec{T})$. The obtained scalar quantity is proportional with the cosine of the $\cos(\vec{S}\ ,\ \vec{T})$.	4 (6.06)	4 /12
PU	. The result of scalar product is ordered from the biggest to the smallest. But, the result does not explain. . The result of scalar product is ordered from the biggest to the smallest, the result depends on the angle between two vectors.	15 (22.73)	16 / 44
MU	. The magnitudes of scalar products of two vectors can not be ordered truly. . The each value that is obtained the scalar product of any two vectors are equal the others.	36 (54.54)	67 / 19
NU	. Non-encodable or unanswered	11 (16.67)	14 / 25

Table 4 shows the ratio of 54.54% incorrect answers to the question, which the teacher candidate numbered [F_3] answered as: "If their magnitudes are equal, their scalar product is also equal. The scheme with scalar product or inner product in the teacher candidate coded [F_3]' memory is faulty. 11 teacher candidates did not answer this question.

Theme 5: Vector Product

Given any two vectors **A** and **B**, the vector product **AxB** is defined as a vector, which has a magnitude of |**A**||**B**|sin(**A**, **B**), where sin(**A**, **B**) is the angle between **A** and **B**.

Table 5. Analysis of the first question themed vector product

Comprehension Level	Encoding	f(%)	Female/ Male(%)
FU	The direction of the vector $\vec{M}\ x\vec{N}$ is drawn as perpendicular to the plane that contains both $\vec{M}$ and $\vec{N}$.	-	- /-
PU	The vector direction is determined as perpendicular, but its sense is opposite.	1 (1.52)	2 / -
MU	It is drawn perpendicular vector. But it is named as the magnitude of vector product. . A vector is drawn in plane. . It is calculated the magnitude of vector product. . The vector product of two vectors is expressed that is equal to the product of the magnitudes of two vectors and the cosine of the angle between them.	32 (48.48)	52 / 38
NU	. Non-encodable or unanswered	33 (50)	46 / 62

Table 5, it is seen the level of understanding and frequencies about the vector **MxN**, where the direction of vector is to use the right-hand rule. The four fingers of the right hand are pointed along **M** and then "wrapped" into **N** through the angle between **M** and **N**. The direction of the upright thumb is the direction of vector. None of the teacher-candidates were able to achieve the correct vector direction to be obtained from the vector product. The geometric significance of the outer product **MΛN** should also be familiar from the standard vector product **MxN** which is not commutativity (Hestenes, 2002). Only one of the teacher-candidates was able to draw the orientation of the vector, but displayed a conceptual delusion. She identified the vector direction incorrectly. The vector product is not commutative and the order in which two vectors are multiplied in across product is important. The teacher candidate numbered [F_5] answered incorrectly the question given as "...**draw the vector:**"||**M.N**||.cos**Q**" (as shown Fig. 3). The teacher candidate coded [F_5] is tried to use knowledge of algebra. This product should have also known as outer product with helping algebra curriculum. Unfortunately, both the schema of vector product and the transfer of algebra knowledge do not occur as expected. However, 33 teacher candidates did not answer this question.

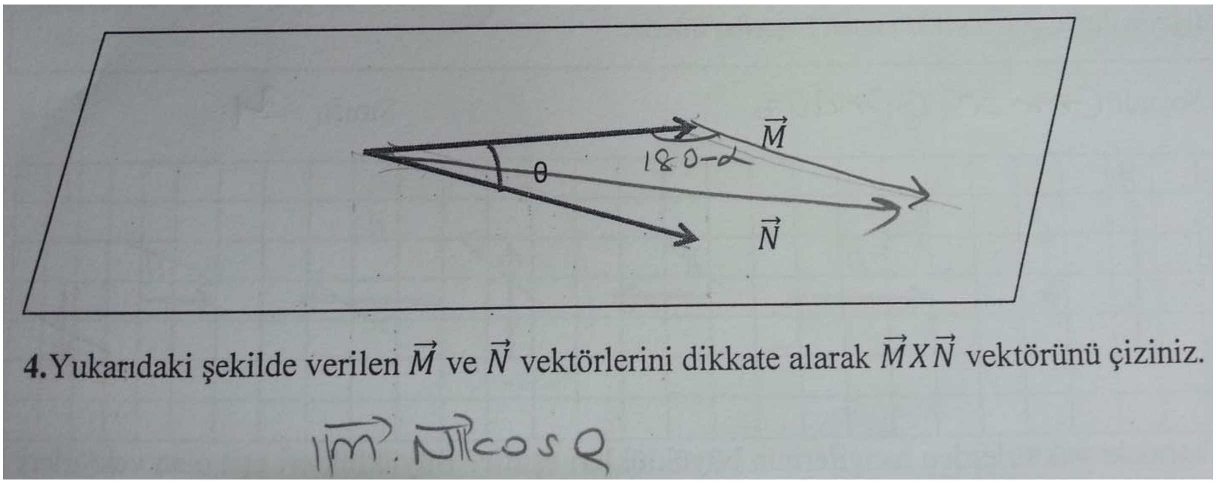

Figure 3. A misunderstanding example of the teacher candidate response to the vector product.

Table 6. Analysis of the second question themed vector product

Comprehension Level		Encoding	f	Female/ Male (%)
FU		. $\lvert\vec{S}x\vec{V}\rvert > \lvert\vec{S}x\vec{U}\rvert > \lvert\vec{S}x\vec{T}\rvert$ is expressed, since $\lvert\vec{S}x\vec{V}\rvert = \lvert\vec{S}\rvert\lvert\vec{V}\rvert \sin(\vec{S}, \vec{V})$. The obtained magnitude of vector product is proportional with the sine of the angle.	1 (1.51)	2 / -
PU		. The results of vector product are ordered from the biggest to the smallest. But, the result does not explain. . The result of vector product is ordered from the biggest to the smallest, the result depends on the angle between two vectors.	7 (10.61)	12 / 6
MU		. The magnitudes of vector products can not be ordered truly. . The each value that is obtained from the vector product of any two vectors are equal the others. The magnitude of vector product is proportional with the cosine of the angle between two vectors. This operation is an inner product.	38 (57.58)	62 / 44
NU		. Non-encodable or unanswered	20 (30.30)	24 / 50

Table 6, it is showed details based on level of understanding of responses given to the question that examines vector magnitude relying on angle to be obtained from vector product. The form of vector product employs the sine of the included angle instead of the cosine. It can be seen that one student answered the question with full understanding. With partial understanding, the question was answered with the rate of 10.61%. Student numbered [F_7] answered the question with partial understanding as " |**SxV**| > |**SxU**| > |**SxT**| *the angle in between is increased* ", displaying a delusion about the quantity of vector product increasing with itself, not with the sine of the angle. The conventional vector product **AxB** is implicitly defined as the dual of the outer product (Hestenes, 2002). It is expressed outer product **M**Λ**N** = i|**M**||**N**|sinθ by Hestenes (2002). Unfortunately, it could not accomplish high road transfer defined Salomon and Perkins (1989). 57.58% of the teacher-candidates gave incorrect answers to this question. Student coded [F_6] gave an incorrect answer to this question as " *the lengths of* |**SxT**| > |**SxU**| > |**SxV**| *would be square of the resultant vector, the one with the highest value will be the longest*" 30.30% of the teacher-candidates were not able to answer this question.

Discussion and Conclusion

In this study, level of understanding and misconceptions of math teacher candidates were revealed on concepts of vector quantities, vector addition, subtraction, scalar and vector product. Accordingly, only 12.12% of the teacher candidates were able to fully express vectors equal in magnitudes. We found that a significant proportion of math teacher candidates had serious conceptual confusion related to the representation of vector magnitude for the first question. This problem was obtained from Nguyen & Meltzer (2003). They found that 63%-87% of students were able to answer the problem correctly. It can be decided that the reason of serious difference is due to the fact that the representation of vector magnitude is given with brackets in the question by Nguyen & Meltzer (2003). For the question about vector addition, teacher candidates graphically opted addition through the triangle law. It was generally observed with misconceptions in partial understanding that students had difficulties while identifying resultant vector, and either reverse-identifying or not even identifying the resultant vector direction. Küçüközer (2009) reports that 44% of students from the programme of primary school teacher education give a correct response to vector addition question. This result is smaller than the 77.27% correct response in our study. Another comparison we may make is to the results reported by Küçüközer on the alternative conceptions involving vector addition. Küçüközer emphasize that 25% of students has alternative conceptions, while the responses of math teacher candidates on misunderstanding level were 7.58% for this paper. For the question regarding vector subtraction, teacher candidates provided correct answers with the rate of 65.15%. It was also observed that the success rate of 77.27% by the teacher candidates in two-dimensional vector addition was higher when compared to the results of studies by Flores et al. (2004), Knight (1995), Küçüközer (2009) and Nguyen & Meltzer (2003). For male teacher candidates, the success rates in vector properties with full understanding are lower than female teacher candidates.It can be said that the results are in accordance with some studies (Flores et al., 2004; Knight, 1995; Küçüközer, 2009; Nguyen & Meltzer, 2003).

In this study, it was expected from participants to assign the results the scalar and vector product through three open-ended questions designed by researchers. Zavala & Barniol designed different opened-ended questions to investigate the difficulties on the calculation and misconceptions in the interpretation of the dot and cross products. In addition, we investigated both knowledge level and gender factor on these two products. As a result, it was observed that the mathematics teacher candidates showed a little success in vector and scalar products (1.51% and 6.06%, respectively). The success rate of math teacher candidates in this study is observed to be significantly lower when compared to the study about vector products conducted by Zavala & Barniol (2010). Female teacher candidates have more misconceptions with dot and vector products than male teacher candidates, but they have smaller rate with non-encodable or unanswered. Teaching techniques and educational materials should be developed to eliminate teacher candidates' failure in vector properties and operations and a better understanding of the concept.

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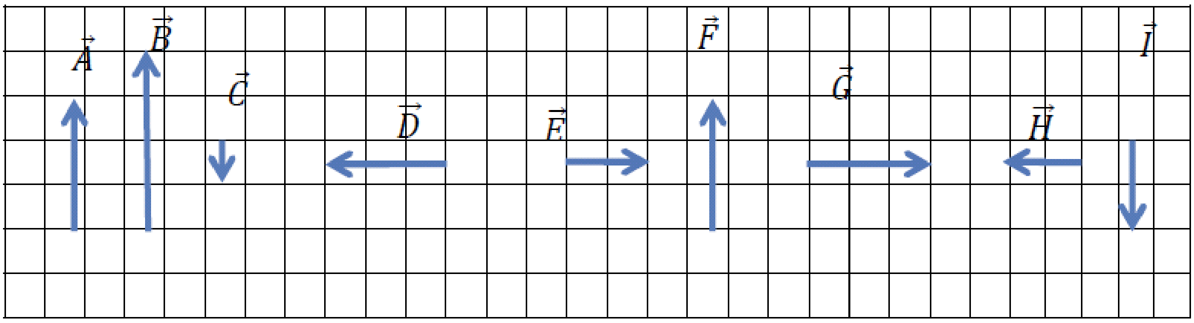
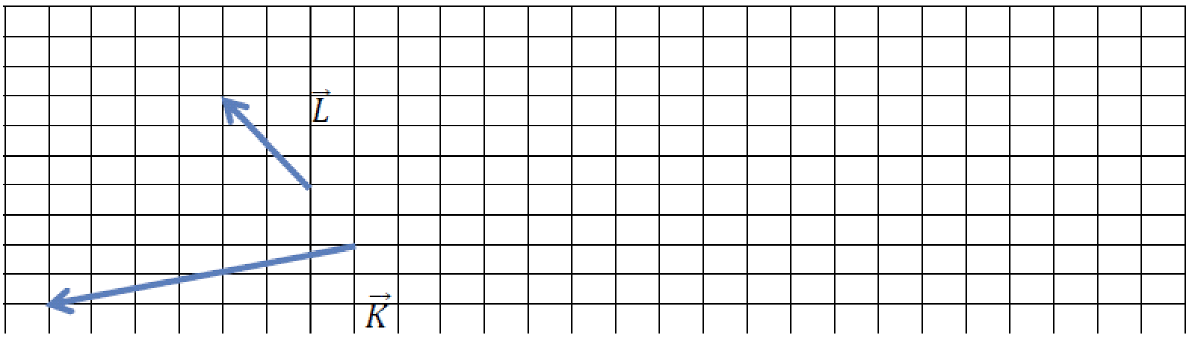
Appendix

Dear teachers, this test aims to evaluate your knowledge on ***"Vectors".*** The answers you provide will not have any effect on your course grades. Your answer sheets will be used within the research being conducted, and your name will be kept confidential in accordance with the codes of ethics. For validity of the research, it is very important that you answer all questions. Therefore, please do not leave any question blank. Thank you for your interest, and good luck.

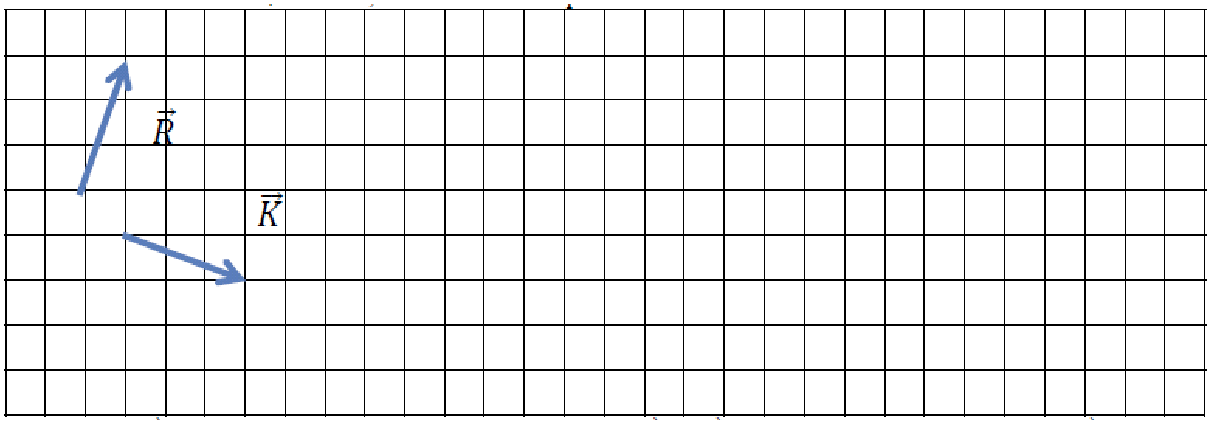
Name & Surname: Class:



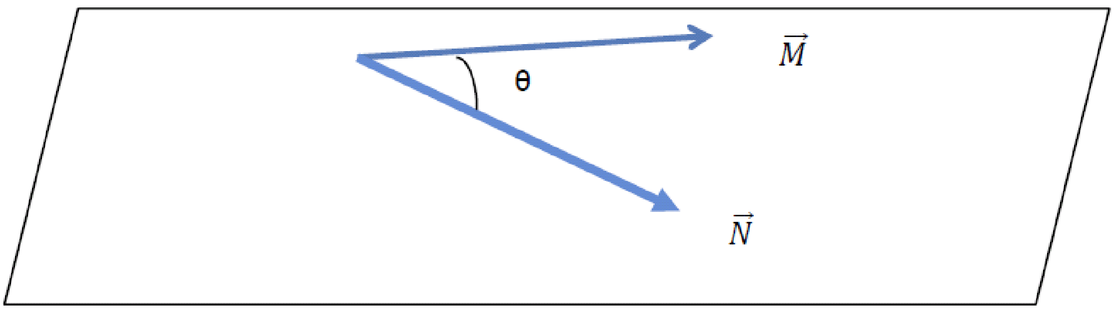
1. Which of the vectors above are equal in magnitudes? Please write down the vectors that are equal in magnitudes. Explain your answer.



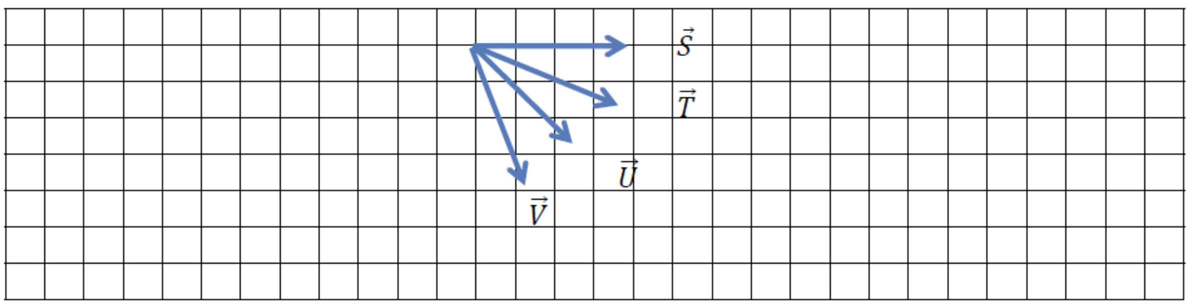
2. Please draw the $\boldsymbol{R} = \boldsymbol{K} + \boldsymbol{L}$vector, which is the cross product of ***K*** and ***L.***



3. The vector ***R*** given above is the cross product of the vectors ***K*** and ***L***. In this case, please find the vector ***L*** by drawing.



4. Considering the vectors **M** and **N** given above, please draw the vector **M**x**N**.



5. Please sort the quantities of ***S***.***T***, ***S.U*** and ***S.V*** in descending order for the vectors ***S***, ***T***, ***U*** and ***V*** given above. (|***S***|=|***T***|=|***U***|=|***V***|) Please explain your answer.

6. Considering the figure given in the 5th question, please sort the quantities of $|\boldsymbol{S}\mathrm{x}\boldsymbol{T}|$, $|\boldsymbol{S}\mathrm{x}\boldsymbol{U}|$ and $|\boldsymbol{S}\mathrm{x}\boldsymbol{V}|$ in descending order. ($|\boldsymbol{S}|=|\boldsymbol{T}|=|\boldsymbol{U}|=|\boldsymbol{V}|$) Please explain your answer.

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